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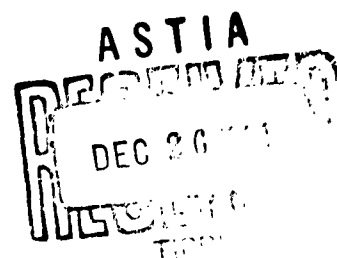
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A Voltage Tunable Oscillator in L-Band

ROBERT D. HALL

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ELECTRONIC DEFENSE LABORATORIES



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A VOLTAGE TUNABLE OSCILLATOR IN L-BAND

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DEFINITION OF SYMBOLS

Symbol	Definition	Unit
B	Susceptance	mhos
c	Capacitance	pf
C_{gp}	Plate-to-grid capacitance of a vacuum tube	pf
C_s	Stray capacitance	pf
C_T	Tuning capacitance	pf
E	Electric field strength	volts/cm
f	Frequency	cycle/sec
G	Conductance	mhos
G_m	Transconductance of a vacuum tube	mho
k	Heat conductivity	cal/(sec/cm ² C)
P	Electric polarization	volts/cm
Q_c	Reciprocal of $\tan \delta$ and equal to B/G	numerical
$\overline{Q_c}$	Average Q of a capacitor	numerical
Q_u	Q of an unloaded resonant circuit	numerical
R	Rate of liberation of heat throughout the volume of a dielectric	cal/sec/cm ³
R'_{FE}	Equivalent circuit resistance corresponding to ferroelectric loss conductance	ohms
R_g	Grid resistance in a vacuum tube circuit	ohms
R_K	Cathode resistance in a vacuum tube circuit	ohms

DEFINITION OF SYMBOLS -- Continued

Symbol	Definition	Unit
R_L	Plate tank load resistance	ohms
R_p	Series plate limiting resistance	ohms
r_p	Vacuum tube plate resistance	ohms
R_u	Unloaded plate tank resistance	ohms
T	Temperature	degrees Kelvin
T_c	Curie Temperature	degrees Kelvin
X_c	Capacitive reactance	ohms
X_L	Inductive reactance	ohms
Z_c	Characteristic impedance of a transmission line	ohms
β	Phase constant of a transmission line	rad/cm
$\tan \delta$	Dielectric loss tangent = ϵ''/ϵ'	numerical
ϵ'	Real part of the dielectric constant	numerical
ϵ''	Imaginary (loss) part of dielectric constant	numerical
ρ	Ratio of the maximum value of a variable capacitance to its maximum value	numerical
ω	Angular frequency equal to $2\pi f$	rad/cm

A VOLTAGE TUNABLE OSCILLATOR IN L-BAND

Robert D. Hall

1. ABSTRACT

A 500-Mc oscillator was constructed using a planar triode tube. The oscillator could be tuned electrically over a 15 per cent bandwidth. The tuning was accomplished by a solid element with low-power drain and no moving parts. The report includes a discussion of the tuning element, tuning technique, circuit design, limitations and some possible improvements.

2. INTRODUCTION.

This report covers the development of a voltage-tunable planar-triode oscillator in the frequency range between 500 and 1000 Mc. The purpose of the work was to assess the capabilities of ferroelectric materials as tuning elements. The frequency range was chosen in response to a general requirement for L-band voltage-tunable oscillators and to demonstrate the ability of ferroelectrics to operate at frequencies beyond the useful range of available diode tuning capacitors.

Tuning is accomplished by varying a ferroelectric capacitor in series with a distributed circuit inductance. The change in capacitance in ferroelectric materials is accomplished by varying a bias voltage applied to the ferroelectric tuning element. Since the tuning element is very small compared to the wavelengths used, it appears as a true lumped element in the circuit.

An oscillator was designed which provided about 100 mw of power over the 500- to 1000-Mc band when it was tuned by low-loss capacitors. When ferroelectric tuning elements were used, the tuning range was 15 per cent and the power output 1 to 5 mw. Design changes mentioned in the text would provide a moderate increase in power output and tuning range. However, the performance of ferroelectric tuning elements in this oscillator is an indication of the quality of available ferroelectric materials for microwave applications.

2. - - Continued.

Although in many cases, microwave components could be designed to overcome most of the limitations of ferroelectrics, the advantage of electrical circuit control thus achieved must be measured against the increased circuit complexity required.

The body of the report includes a discussion of ferroelectrics and ferroelectric tuning, the design of the oscillator, experimental results and conclusions. Mathematical details are arranged in the appendixes which follow.

3. FERROELECTRIC TUNING.

3.1 Ferroelectrics.

3.1.1 Background. Ferroelectrics are a complex class of dielectric materials discovered around the beginning of the century. They became objects of scientific interest because they are piezoelectric and because the dielectric constant varies with temperature, frequency, and electric field strength. Applications of piezoelectrics as electromechanical transducers began a trend of engineering interest in ferroelectrics which continued to develop as computer applications were sought. More than 70 ferroelectric materials are now known and their properties are being intensively studied.

3.1.2 Definitions. A ferroelectric is a crystalline, or ceramic, material. Below some temperature it exhibits a hysteresis of polarization with applied bias. The hysteresis curve on a plot of polarization vs. electric field (Figure 1), appears similar to a curve of magnetization vs. magnetic field and led to the naming of ferroelectrics by analogy. Ferroelectricity, however, has so little in common with ferromagnetism that no further mention of it need be made.

Ferroelectrics have dielectric hysteresis because they have spontaneous polarization. That is, below a transition temperature, opposite faces of a ferroelectric crystal become oppositely charged in the absence of any applied polarizing field. The transition temperature is called the "Curie Temperature".

In terms of crystal structure, a Curie temperature is a temperature at which the atoms of a crystal rearrange so that the spatial symmetry of the crystal is changed. There are crystals in which such changes in crystal symmetry occur at more than one temperature. In ferroelectrics, a change in the state of polarization of the crystal also occurs

3. 1. 2 - - Continued.

at the Curie temperature. In crystals with one Curie temperature, spontaneous polarization disappears above the Curie temperature. Crystals with more than one Curie temperature may be ferroelectric only between two Curie temperatures, as in the case of Rochelle salt, or may have more than one polarized state in different temperature ranges, as in the case of barium titanate. Most titanate ferroelectrics are more readily obtained in ceramic form than in the form of single crystals. Indeed, some ferroelectrics have never been prepared artificially in sizes larger than microscopic.

3. 1. 3 The Dielectric Constant. The dielectric constant of a ferroelectric crystal, or ceramic, has the characteristic temperature variation shown in Figure 2. The dielectric constant may also vary with frequency and applied electric field. The sharp peak in the dielectric constant of a crystal is broadened in some ceramics. The dielectric constant and loss tangent of one ceramic, Glenco K9000, are shown in Figures 3 and 4. The data are given over a temperature range of 25°C. to 85°C. and an electric field range of 0 to 40,000 V/cm, and at 500 Mc. Behavior at 1000 Mc is very similar. Glenco K9000 is a titanate ceramic mixture in which additives have been used to reduce the temperature dependence of the dielectric constant for miniature capacitor applications at frequencies below about 50 Mc. Variability of the dielectric constant with applied electric field is not a desirable property in fixed capacitors and was probably not maximized. Nevertheless, Glenco K9000 is presently one of the best commercially available ferroelectrics for microwave applications. The titanates are only one family of ferroelectrics. In some other families, the dielectric constant may vary as much as 1000 to 1 with temperature. The electric field dependence of the dielectric constant has not been measured for most of these materials.

To use these materials as tuning elements at constant temperature, one need only know what range of capacitance values is required and if the curves show that this range is within the capability of the material, the size of the tuning element may be chosen from the formula for the capacitance of a parallel plate capacitor.

$$C = 0.225 \frac{A}{d} \epsilon' \quad (1)$$

A is the area in square inches

d is the thickness in inches

C is in pf

ϵ' is the real part of the dielectric constant.

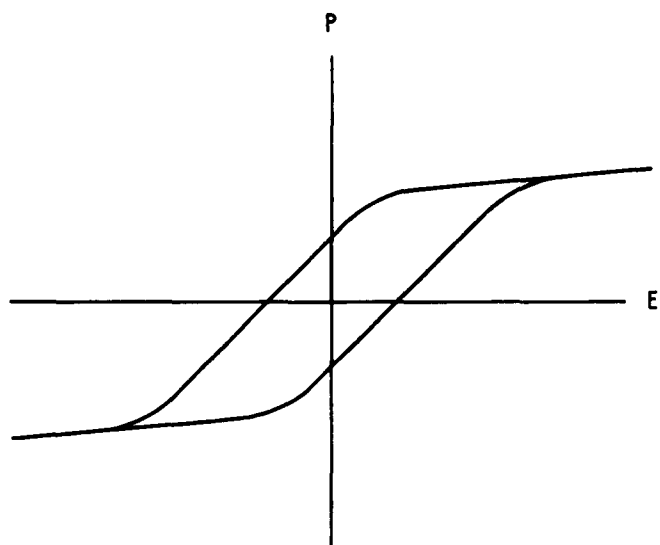


Figure 1

Dielectric Hysteresis

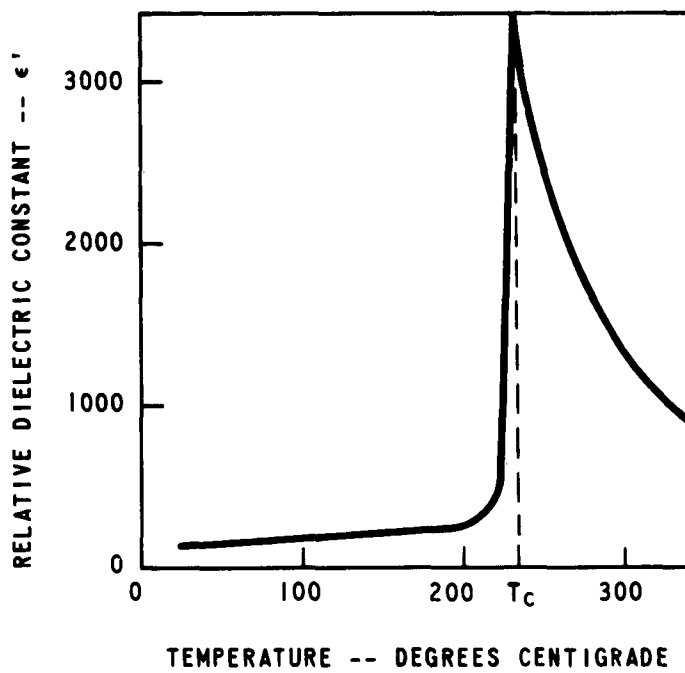


Figure 2

Dielectric Behavior of Ceramic Lead Zirconate

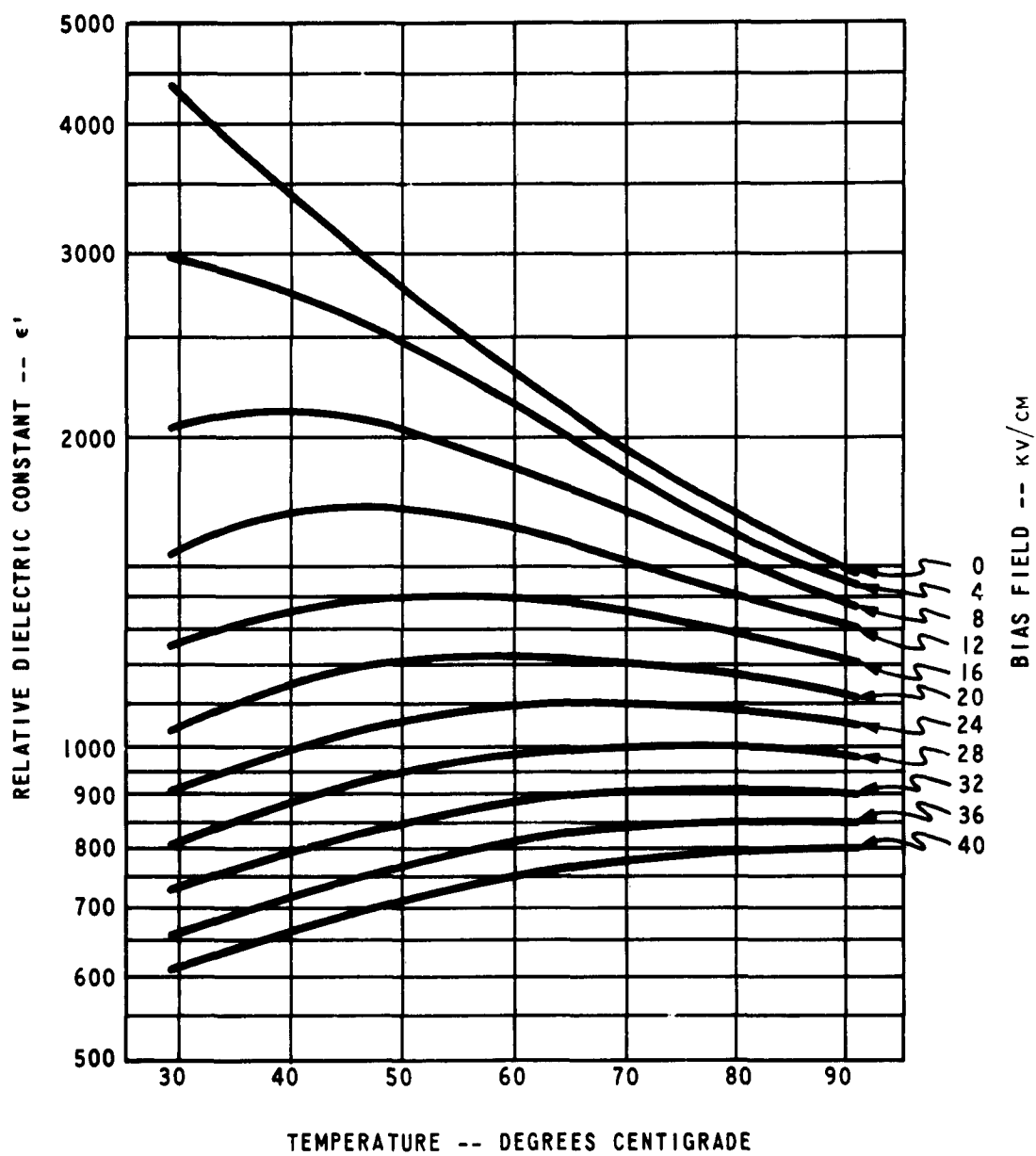


Figure 3

Dielectric Constant of Glenco K9000 at 500 Mc

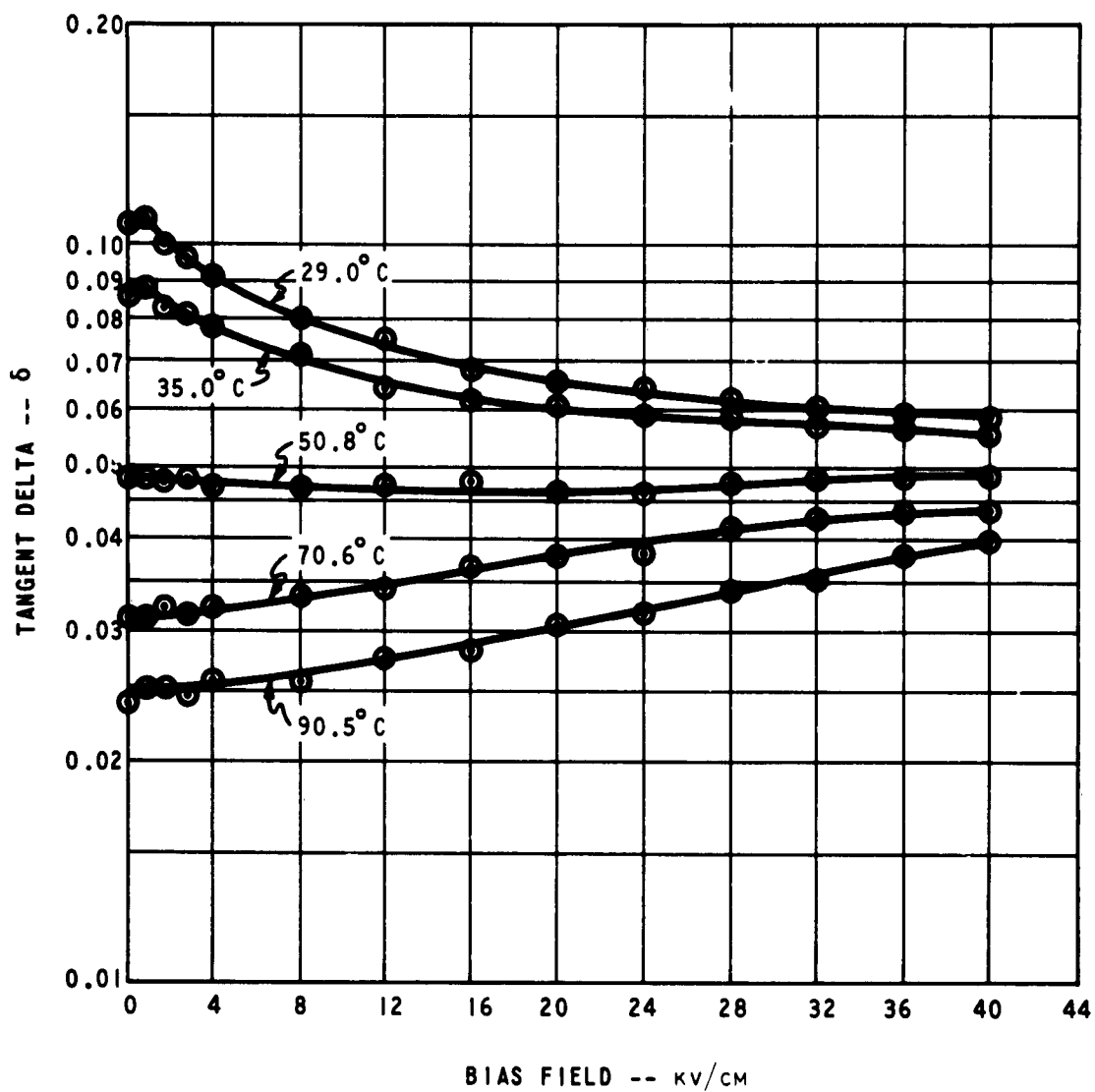


Figure 4

Loss Tangent of Glenco K9000 at 500 Mc

3.1.4 Ferroelectric Losses. Various mechanisms combine in a ferroelectric to produce dielectric losses. These losses reduce the Q of the circuit in which the ferroelectric is used to an extent which depends on the Q of the circuit without the ferroelectric and on the fraction of the total electric field energy which is stored in the ferroelectric. When a high circuit Q is desired, low-loss components must be used and the energy stored in the "lossy" components must be a small fraction of the total stored energy. If a circuit contains n energy storage elements and if Q_i is the Q of the i th element, the total circuit Q is

$$Q = \frac{1}{\sum_{i=1}^n \frac{K_i}{1 + Q_i}} \quad (2)$$

where i counts the energy storage elements and K_i is the fraction of the total energy which is stored in the i th element. The losses in ferroelectrics are greater than those of commonly used dielectrics. At microwave frequencies, the Q of available ferroelectric materials is between 3 and 30, a small fraction of the Q 's of "normal" microwave components. When a resonant tank circuit is to be tuned over a wide range by a ferroelectric, most of the electric energy in the tank must be stored in the ferroelectric. At best, the Q of such a tank circuit will be of the same order as the ferroelectric Q . The development of high Q ferroelectric tuning elements would greatly ease the problems of designing electrically-tuned circuits.

3.1.5 Permissible Heating. The power level in a circuit which uses a ferroelectric will be limited to values at which the ferroelectric is not overheated. The fraction of the power in the circuit which is lost as heat in the ferroelectric will depend upon the amount of energy stored in the ferroelectric. High-power levels are permissible when the ferroelectrics are used as bypass capacitors, because little energy is stored in the ferroelectric and its exact capacitance value is unimportant. On the other hand, ferroelectrics which are used for wide-band tuning must operate over a narrow temperature range which definitely limits the power level. The permissible power level in the circuit is determined by the amount of power dissipated in the ferroelectric. If the ferroelectric is mounted between two metal posts as in Figure 5, which act as constant temperature heat sinks, and heat is generated in the volume of the dielectric at the rate R cal/sec/cm³, we can calculate the minimum equilibrium temperature. Assuming that very little heat is lost from the sides of the ferroelectric compared

3.1.5 - - Continued.

to the heat conducted away at the ends, then the equation for the maximum temperature in the ferroelectric can be derived (see Appendix A).

$$T_{\max} = \frac{R l^2}{8 k} \quad (3)$$

where l is the length of the ferroelectric

k is the heat conductivity of the ferroelectric.

For barium titanate, this becomes

$$T_{\max} = \frac{R l^2}{24 \cdot 10^{-3}} \quad (4)$$

When this equation is applied to the practical case of a ceramic tuning element having the dimensions of a cube with an edge length of 0.010 inch, the result is that

$$\frac{T_{\max}}{R} = \frac{1.15^{\circ} \text{C}}{\text{mw}} \quad (5)$$

This means that one milliwatt of power dissipated in the ferroelectric will raise the temperature at least 1.15°C . The rate, R , of energy dissipation in the ferroelectric depends upon its Q and placement in the circuit. The result given represents a lower limit for the temperature rise because, although some cooling is obtained from the sides as well as from the ends of the ferroelectric, this cooling is overcome by temperature rise of the metal end posts. In practical applications, they cannot be considered as heat sinks. Other sources of heat, such as vacuum tubes, may also raise the ambient temperature of the ferroelectric.

3.1.6 Hysteresis and Aging. Some ferroelectrics exhibit two effects which must be considered by the designer. Neither of these is well understood. If a sequence of voltages is applied to a ferroelectric ceramic the response of the dielectric constant is not instantaneous when the applied bias is switched. The length of time required for the dielectric constant to assume its stable value after the bias is switched varies with temperature, bias voltage, and composition. It usually is stable in less than a few minutes and in some cases, microseconds. The second effect reported in the literature, is the gradual change with time on a scale of days, months, or even years of the properties of ferroelectrics. The ceramic forms seem to be particularly susceptible. The dielectric constant of titanate capacitor

3.1.6 - - Continued.

materials is especially subject to aging. Whether all ferroelectrics age is not known, but aging, hysteresis, and lack of a threshold field for switching have made impractical the use of known materials in computer memories.

3.2 Construction of Tuning Elements.

3.2.1 Raw Materials. Ferroelectric crystals are hard, fragile, and in some cases unstable. Some of the crystals have cleavage planes, and when this is the case, the ferroelectric direction is either normal or parallel to the cleavage plane. This means that the crystalline direction in which the zero bias dielectric constant has its maximum value, is also parallel or normal to the cleavage plane. The bias voltage is usually applied along the direction of maximum zero bias dielectric constant in order to achieve maximum capacitance variation. Ceramic ferroelectric materials may be isotropic or oriented and are usually available in the form of thin discs silvered on the two plane sides.

3.2.2 Element Size. The final size of the ferroelectric tuning element is determined by the capacitance and bias requirements. The capacitance depends upon the dimensions according to the formula of Section 3.1.3.

Because the dielectric constant for the best available ferroelectrics is high ($500 \leq \epsilon < 5000$), the size must be very small. Moreover, to realize maximum variation, the bias potential must reach 100 to 200 volts/mil (0.001 inch), of thickness. For a potential of 1000 volts, the thickness is therefore restricted to 0.005 to 0.010 inch. The area of the electrode surfaces must be made correspondingly small to keep the capacitance in the desired range. Typically, capacitance values useful in tuning microwave circuits will be between 0.1 and 10 pf. Using Hi Q 91 Aerovox ceramic with a zero bias dielectric constant of about 4500, a tuning capacitor with a zero bias capacitance of 10 pf is a 0.010-inch cube silverplated on two opposite faces.

3.2.3 Capacitor Preparation. To prepare a ferroelectric tuning element, the raw material is cut, or ground, into discs of the required thickness and silverplated on both sides. The plating can be fired or vacuum deposited, by high temperature evaporation. The discs are cut into strips of the desired width by a diamond wheel. Finally, the strips are sectioned into cubes also using a diamond wheel.

To assure adherence of the silver and to avoid chipping the corners of the cubical tuning capacitors during the cutting operations, all cutting

3.2.3 - - Continued.

is done with the workpiece surrounded by a hard cement and mounted between ceramic plates. In the first operation, that of cutting the disk into strips, the disk is cemented between ceramic plates using a stick shellac-type cement. In each pass, the diamond wheel is made to cut through the top coverplate and through the workpiece, but only into and not through the bottom ceramic plate. The ferroelectric strips are removed from the ridges of the baseplate which are created during the first cutting operation, and dropped into the slots between the ridges. A new coverplate is used and again a stick shellac cement holds the strips in the slots and the coverplate in place, while cuts are made perpendicular to the first cuts. The ferroelectric cubes are removed from the ceramic baseplate matrix and then cleaned, ready for mounting.

3.2.4 Mounting. The tuning capacitors are too small and delicate to use unmounted; yet the package in which they are mounted must add a minimum of stray shunt capacitance and series inductance. The difficulties in mounting ferroelectrics were overcome by the design shown in Figure 6. The cartridge in Figure 6 is shown about three times actual size. The outer case is a Rexolite 1422 plastic tube with a transverse hole. The ferroelectric tuning capacitor is soldered to a metal post which is push-fitted into one end of the Rexolite body. A flexible metal flap is soldered to the end of a similar post and pushed into the opposite end of the body until the flap contacts the silvered surface of the capacitor. The flap is soldered to the silvered surface and insulating compound is injected through the lateral hole in the Rexolite body until the tuning capacitor and its contacts are completely covered. The compound prevents dc breakdown at the high-bias potentials used.

3.3 Tuned Circuit Design.

3.3.1 Shorted Transmission Lines. The input impedance of a shorted line is inductive at frequencies where its length is less than a quarter wavelength. At such frequencies, the line can be tuned to resonance with an appropriate capacitance. Changing the capacitance varies the resonant frequency in a manner similar to the tuning of a lumped element tank circuit, as in Figure 7. The frequency ratio over which a variable capacitor will tune a lumped inductance is exactly the square root of the capacitance ratio. However, the frequency ratio over which the same capacitor will tune a distributed inductance depends upon how nearly the maximum frequency approaches the self-resonant (quarter wavelength) frequency of the distributed inductance. As the maximum frequency to be tuned approaches the self-resonant frequency, the required tuning capacitance approaches zero. Since the range of

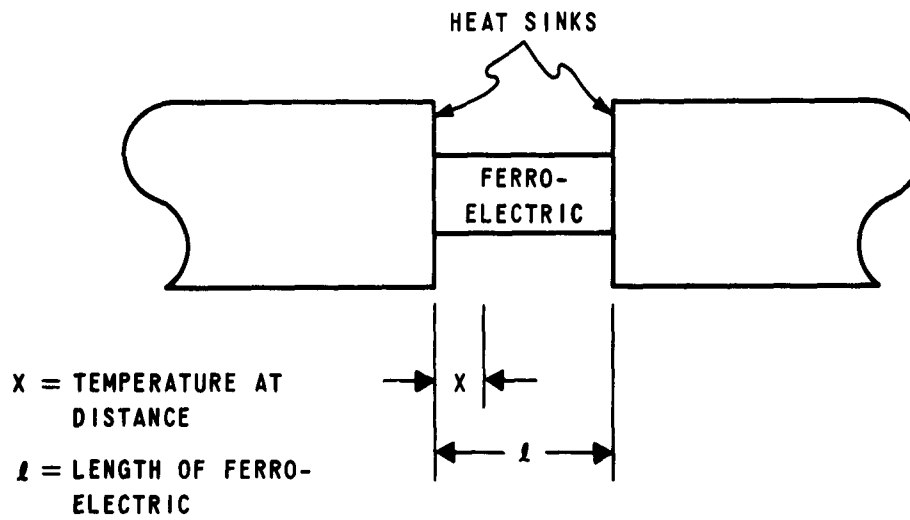


Figure 5

Ferroelectric Between Heat Sinks

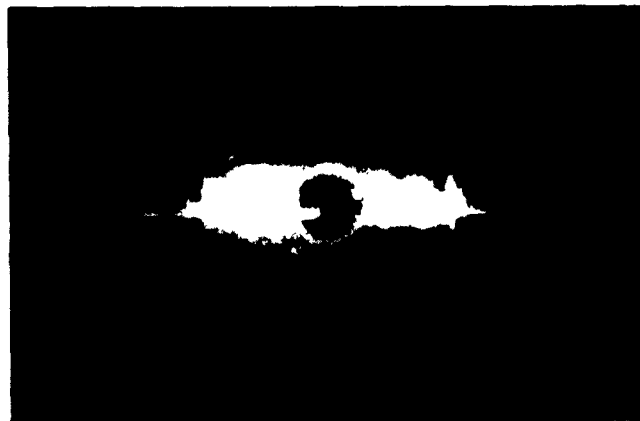


Figure 6

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Ferroelectric Cartridge

3.3.1 - - Continued.

variation of the capacitor, $\rho = C_{\max}/C_{\min}$, is usually limited to about 10, and since near the self-resonant frequency the inductive reactance increases very rapidly, very little change in frequency can be achieved near the self-resonant frequency because the required C_{\min} goes to zero. The distributed inductance approaches the tuning characteristic of a lumped inductance when it is very short compared to the wavelength. The Q of a lightly-loaded tuned cavity will usually be the same as the $Q_c = B/G$ of the capacitor since transmission line cavity losses can usually be made quite small.

3.3.2 Capacitance Loaded Lines - Shunt Tuning. When the line is loaded by having a capacitance instead of a short circuit at one end, there are two possible tuning methods. The tuning capacitor can be placed across the line in parallel with the loading capacitance, or in series with the line and loading capacitance. For example, capacitive loading of coaxial lines inherently occurs at the grid-cathode and grid-plate gaps of a planar triode, and must be considered in high-frequency oscillator design.

The parallel case, with the tuning capacitance very close to the loading capacitance as in Figure 8, is very similar to a shorted line. The effective tuning capacitance ratio is simply reduced by the presence of a fixed shunt capacitance. The differences are that an increased tuning capacitance ratio is required for a given frequency ratio, and, if the stray capacitance has a higher Q than the tuning capacitor, the Q is increased. If the required capacitance ratio to tune the frequency range desired is ρ , then

$$\rho C_{\min} = C_{\max} \quad (6)$$

$$\text{but } C_{\min} = C_{\text{stray}} + C_{T \min} \quad C_{T \min} = \text{min. tuning capacitance.} \quad (7)$$

$$C_{\max} = C_{\text{stray}} + C_{T \max} \quad C_{T \max} = \text{max. tuning capacitance.} \quad (8)$$

$$\text{Hence, } \rho(C_{\text{stray}} + C_{T \min}) = C_{\text{stray}} + C_{T \max} \quad (9)$$

and

$$(\rho - 1) C_{\text{stray}} = C_{T \max} - \rho C_{T \min} \quad (10)$$

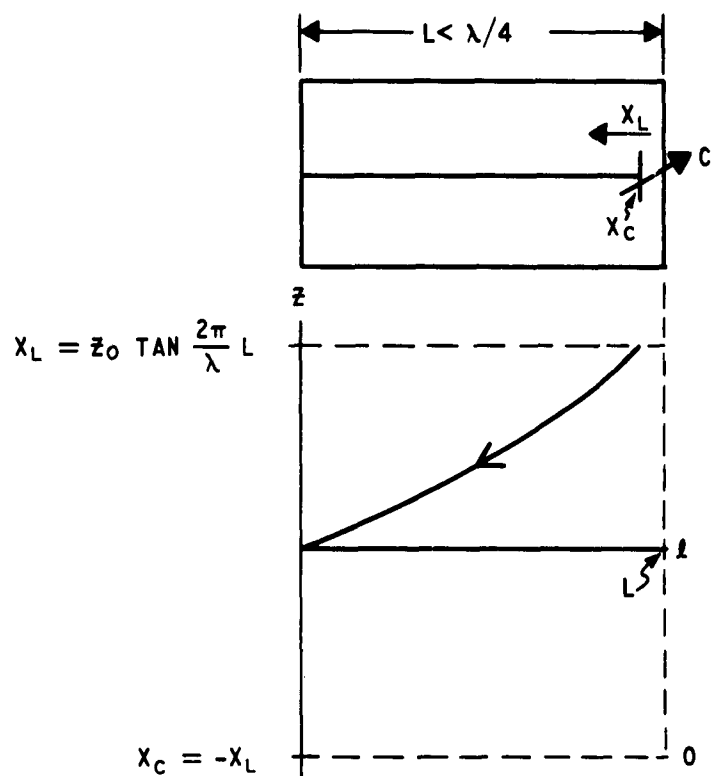


Figure 7

Capacitive Cavity Tuning

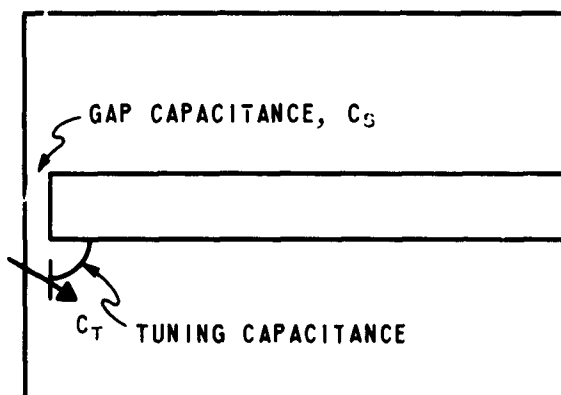


Figure 8

Parallel Tuned Capacitance Loaded Cavity

3.3.2 - - Continued.

$$\frac{(\rho - 1)C_{\text{stray}}}{C_{T \min}} = \frac{C_{T \max}}{C_{T \min}} - \rho, \quad (11)$$

$$\frac{C_{T \max}}{C_{T \min}} = \rho' = \rho + (\rho - 1) \frac{C_{\text{stray}}}{C_{T \min}}. \quad (12)$$

The Q of the circuit is increased if the stray or loading capacitance has a Q much higher than that of the ferroelectric. This is usually true in the plate tank circuit of planar triode oscillators, but not in the grid circuit. The Q_c of a capacitor is defined with reference to the diagram as

$$Q_c = \frac{B}{G} = \frac{\omega C}{G} \quad (13)$$

where

C is the equivalent parallel capacitance
 G is the equivalent loss conductance.

In the parallel circuit, the circuit Q is

$$Q = \frac{B}{G} = \frac{B_{FE} + B_{\text{stray}}}{G_{FE} + G_{\text{stray}}}. \quad (14)$$

If $G_{\text{stray}} \ll G_{FE}$ as is often true, then in the previous equation

G_{stray} may be ignored with the result that

$$Q = \frac{B_{FE}}{G_{FE}} + \frac{B_{\text{stray}}}{B_{FE}} = Q_{cFE} + \frac{B_{\text{stray}}}{B_{FE}}. \quad (15)$$

That is, the circuit Q is increased from the ferroelectric Q_c by the ratio of stray shunt susceptance to ferroelectric loss conductance.

Alternatively,

$$Q = Q_c \left(1 + \frac{B_{\text{stray}}}{B_{FE}} \right) = Q_c \left(1 + \frac{C_s}{C_{FE}} \right). \quad (16)$$

The increase in Q is greater at the high frequency end of the tuning range where C_s/C_{FE} is largest.

3.3.3 Capacitively Loaded Lines - Series Tuning. At a particular frequency, the lumped equivalent for a distributed circuit is useful in calculating the Q . The effects of changing the resonant frequency, however, must be calculated from the proper expressions for the distributed parameters involved. Therefore, the discussion of series-tuned lines falls naturally into two parts, the calculation of circuit Q and the placement of the tuning capacitor to obtain maximum tuning range.

3.3.3.1 The Q of a Series-Tuned Circuit. The distributed circuit of Figure 9 can be represented at a fixed frequency by the lumped equivalent, Figure 10. The Q of this circuit can be derived (Appendix B), and is given by the formula

$$Q = Q_c \frac{C_T}{C_s} \quad (17)$$

Since C_T and C_s are in series and since if two capacitors of widely differing values are placed in series, the resultant capacitance is approximately equal to the smaller, C_T must vary from approximate equality with C_s to values much smaller than C_s in order to cover a wide frequency range. The formula shows that the circuit Q will be substantially less than the ferroelectric Q at the upper frequencies of a wide-range tuned circuit of the type shown in Figure 9.

3.3.3.2 Placement of the Capacitive Tuning Element for Maximum Frequency range on a Distributed Capacitive Loaded Line.

It is not always possible to use a parallel-tuning arrangement. Many microwave planar triodes have large glass envelopes which effectively prevent the ferroelectric tuning element from being placed near the plate-grid gap. If series tuning is necessary, the position of the ferroelectric on the line may be chosen so that the largest possible tuning range is obtained (See Figure 9). The largest tuning ratio with a given capacitance ratio will be obtained when the tuning capacitor is placed where the difference between the highest and lowest reactances reflected from the loading capacitance has its smallest magnitude. These extremes occur at the high and low frequency limits of the tuning range. The principle can be expressed analytically as follows: Let X_1 be the reactance reflected from C_s at a distance of l centimeters at a frequency f_1 ; let X_2 be the corresponding reactance at f_2 ; then the difference between X_1 and X_2 is:

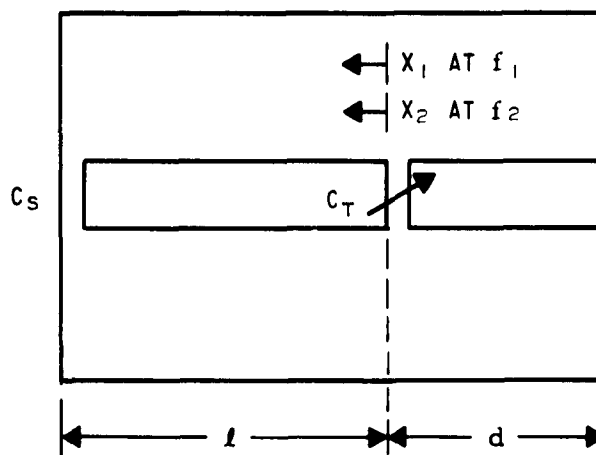


Figure 9

Series Tuned Capacitance Loaded Cavity

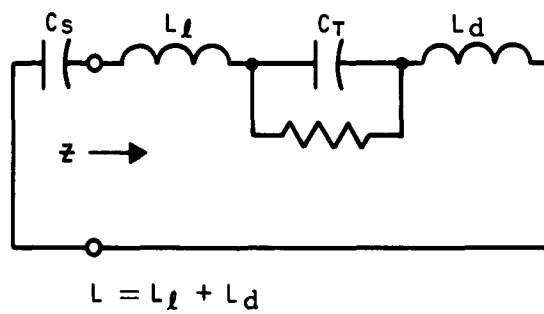


Figure 10

Single Frequency Equivalent Lumped Circuit

3.3.3.2 - - Continued.

$$X = X_1 - X_2 = Z_0 \tan \left[\beta_1 \ell + \tan^{-1} \frac{-1}{\omega_1 C_s Z_0} \right] - Z_0 \tan \left[\beta_2 \ell + \tan^{-1} \frac{-1}{\omega_2 C_s Z_0} \right] \quad (18)$$

The position, ℓ , at which X is extreme can be found as the solution of

$$\frac{dX}{d\ell} = 0 = \frac{Z_0 \beta_1}{\cos^2 \left[\beta_1 \ell + \tan^{-1} \frac{-1}{\omega_1 C_s Z_0} \right]} - \frac{Z_0 \beta_2}{\cos^2 \left[\beta_2 \ell + \tan^{-1} \frac{-1}{\omega_2 C_s Z_0} \right]} \quad (19)$$

Once the position of the tuning element is fixed, the resonant frequency is calculated from the resonance condition. At resonance the total reactance vanishes at every point in the cavity.

The solution of this equation for an individual case may be found graphically. The easiest form to graph is probably

$$\cos \left(r \beta_1 + \tan^{-1} \frac{-1}{r \omega_1 Z_0 C_s} \right) = r \cos \left(\beta_1 + \tan^{-1} \frac{-1}{\omega_1 C_s Z_0} \right) \quad (20)$$

where $r = \frac{\omega_2}{\omega_1}$. Graphs of the two sides of this equation vs. ℓ will intersect at the value of ℓ for which the tuning range will be a maximum.

4. OSCILLATOR DESIGN.4.1 Tube Characteristics.

At low frequencies, high efficiency in oscillators and power amplifiers may be obtained by operating the tube in Class C. For a variety of reasons, some of which are not well understood, Class C operation is not achievable at microwave frequencies. In modern microwave planar triodes, reduction in electrode sizes and spacings is becoming increasingly difficult. Tubes are available which will oscillate up to 10 Gc. As the frequency increases between 100 Mc and 10 Gc, electrode sizes must be reduced with consequent reduction in emission

4.1 - - Continued.

and dissipation ratings, transit time effects increase, grid loading becomes more troublesome, and a premium is placed on transconductance and low-output capacitance.

For the experimental ferroelectrically-tuned oscillator, a tube was needed which would not overheat the ferroelectric element and which could operate at frequencies much higher than intended. The output capacitance should be so low that even with the additional ferroelectric tuning capacitance, the tube would still oscillate in the band of interest. The Sylvania 6BA4, a small glass-envelope planar triode designed for 3 Gc operation, was chosen. It was a good choice among available tubes but planar tubes are still being improved, and better ones are now available.

The electrical parameters of the 6BA4 in the 500 to 1000 Mc range are similar to their low-frequency values except that, as is typical of planar triodes, the transconductance is reduced 10 to 20 per cent. As amplifiers, planar triodes in grid separation circuits with matched input and output will provide about 14-db small signal gain and about 7-db gain at full power output. Their bandwidth is determined in most cases by the plate tank Q since the grid presents an input conductance about equal to the transconductance. The grid-cathode tank is, therefore, a very low Q circuit. In many cases, this fact simplifies tuning and tracking problems.

4.2 Circuit Design.

4.2.1 Type of Oscillator. Most planar triodes operate in one of the circuits in Figures 11 and 12. The former is called a "re-entrant" oscillator and the latter a "grid separation" oscillator. In both circuits, lead inductance is utilized by making the tube electrodes integral parts of the resonators. The re-entrant oscillator simplified the feedback problems by obtaining feedback from the mutual coupling of the grid-cathode line (a) with the grid-plate line (b) through the common region (c). Cavity tuning is usually done by moving the end walls to alter the cavity lengths. The re-entrant oscillator is less frequency stable than the grid separation oscillator, especially if one cavity of the latter is an extra half-wavelength long. Although the re-entrant oscillator can be tuned over a wide frequency range by a single control, the grid separation oscillator was used in the experimental model to obtain frequency stability and because it is easier to analyze.

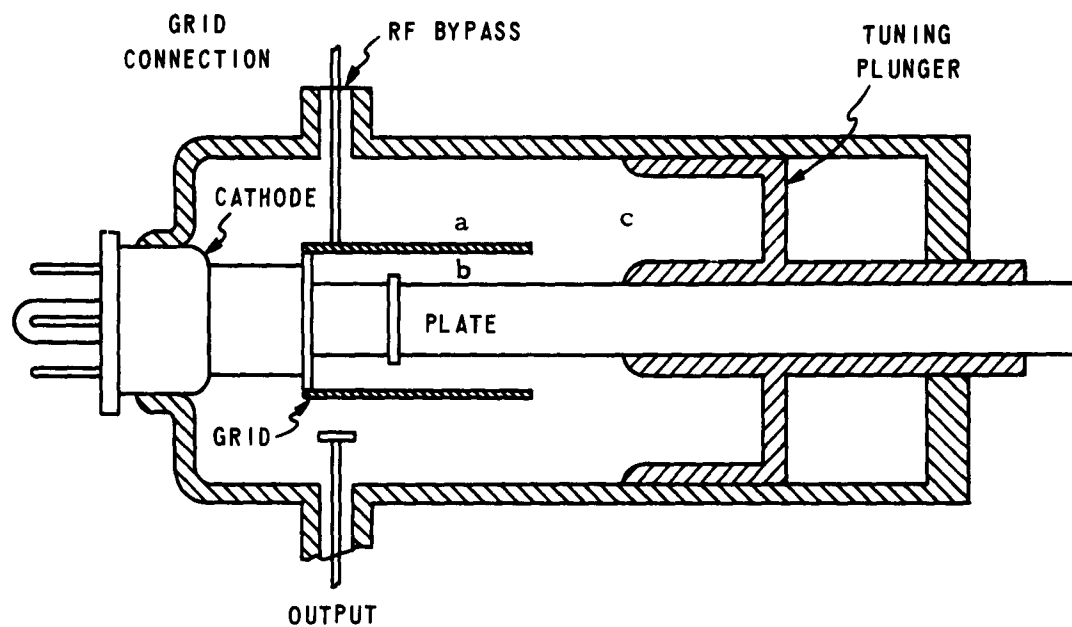


Figure 11

Re-entrant Oscillator

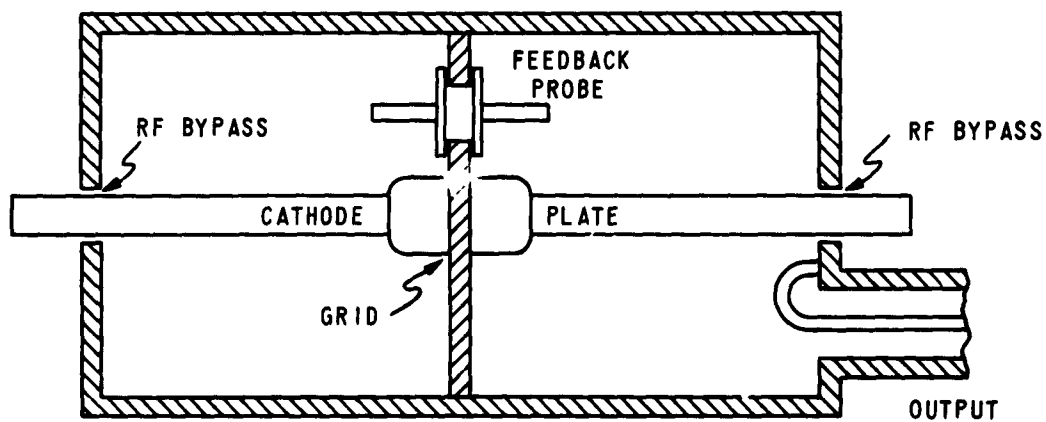


Figure 12

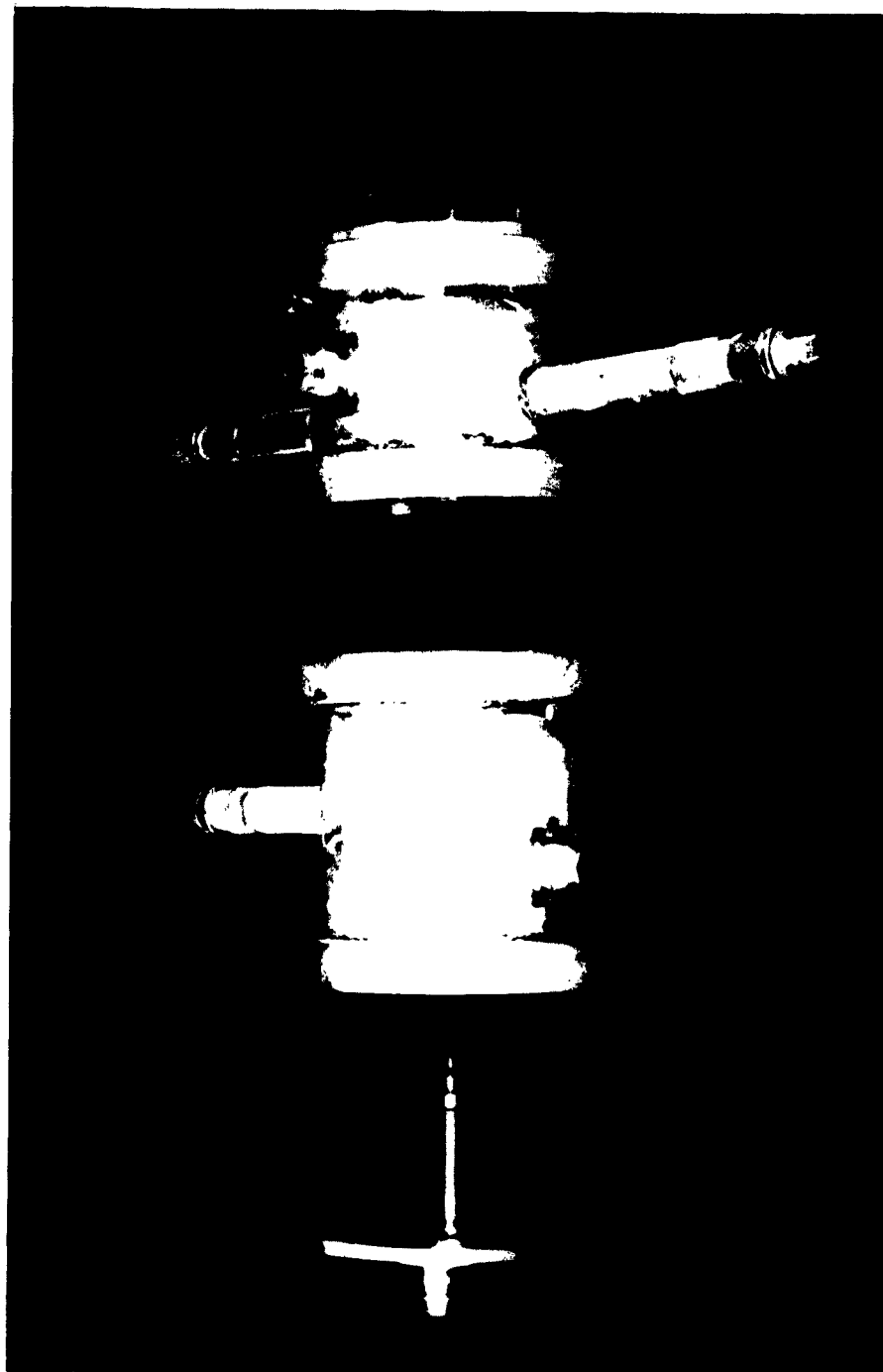
Grid Separation Oscillator

4.2.2 Feedback. Feedback from the plate to the cathode is achieved by a probe, or probes, which pass through holes in the grid plane. The cathode may be connected to the probe by a coin silver strip. That probe feedback operates well over the range from 500 to 1000 Mc was tested by using small fixed teflon tuning capacitors.

4.2.3 Output Coupling. Conventional single-loop output coupling is used from the plate cavity to the load. Additional loops were provided in the cathode and plate cavities so that their resonance characteristics during warm-up could be studied by transmission measurements. The extra type N connectors visible in Figure 13 connect to these loops. They are superfluous in the finished oscillator.

4.2.4 Tuning. The formulas of Section 3 were used to design the tuned circuit. The large glass envelope of the 6BA4 made series tuning necessary. Since the oscillator was designed partially for comparison of various ferroelectric materials, the position of a tuning capacitor which would maximize the tuning range was calculated. It was found to be too close to the ends of the plate and cathode leads for convenience. The ferroelectrics are placed a little farther from their respective electrode gaps with only a small loss of theoretical tuning range, which is given in Figure 14.

4.2.5 Cavity Size. The tuning range of the oscillator is increased by increasing the characteristic impedance of the cavities because the cavities can be made shorter; i. e., more like a lumped inductance. Since the characteristic impedance is proportional to the logarithm of the ratio of conductor diameters, this ratio should be large. However, if the outer conductor is made large enough, undesired higher order modes may interfere with the operation of the oscillator. Moreover, the diameter of the inner conductor cannot be reduced below about 0.25-inch near the tube because it must slip over the plate or cathode stem. The remainder of the inner conductor supports and supplies tuning potentials to the ferroelectric. The increase in characteristic impedance decreases the cavity Q but this effect is negligible. As a practical compromise, the inner conductor diameter was set at 0.25-inch and the outer conductor diameter at 3.5 inches. With these dimensions the characteristic impedance is about 160 ohms. The plate cavity is 10.87-cm long with the tuning capacitor 6 cm from the grid-plate gap and the cathode cavity is 8.37-cm long with the tuning capacitor 4 cm from the grid-cathode gap. The lengths were chosen to obtain operation in the band between 500 and 1000 Mc.



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Figure 13
Partially Assembled Oscillator

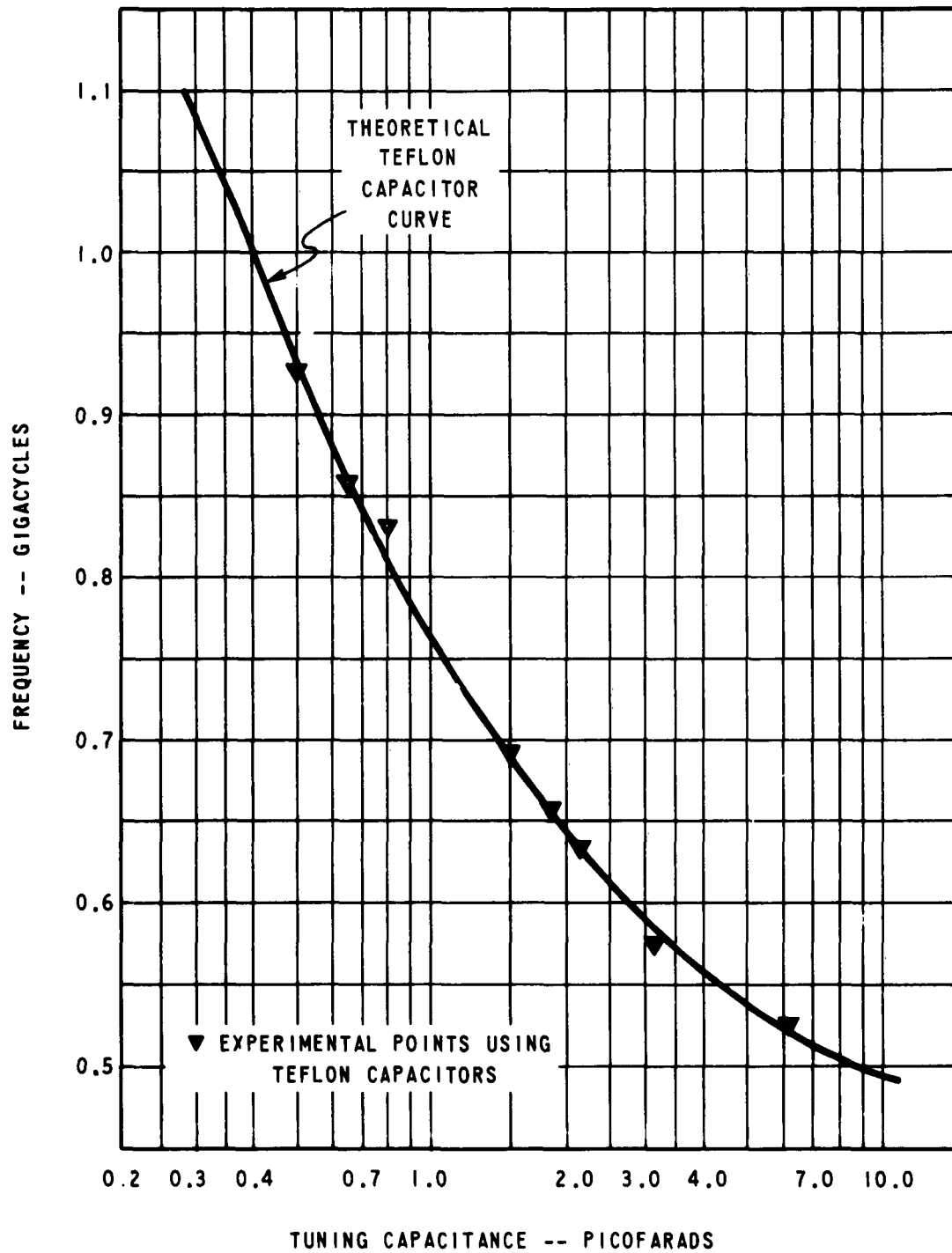


Figure 14

Tuning Curve: Theoretical Compared with Experimental Data

4.2.6 Tuning Curve. The resonant frequency of the plate cavity determines the frequency of the oscillator, because the grid resistance normally reduces the Q of the cathode cavity to $\frac{1}{10}$ or less of the plate cavity Q. As a check on the design, the resonant frequency of the plate cavity was calculated using the formulas of Section 3 and various trial values of tuning capacitance. The result is shown in Figure 14 for comparison with the experimental data in the next section. Based on this curve and ignoring other effects, a variable capacitor would be expected to tune the oscillator from about 650 to 950 Mc. This range is severely reduced by the low Q and large temperature dependence of available ferroelectrics.

4.2.7 Ideal Power Output. With high Q resonant tank circuits, the power output should be nearly independent of frequency and depend only on the load resistance as seen by the tube. For maximum output power, the load resistance should equal the plate resistance. Since the maximum plate current is about 20 ma peak and the plate resistance is about 9000 ohms, the power output should be about 100 mw with a plate voltage of 180 volts.

5. THE EXPERIMENTAL OSCILLATOR.

5.1 Construction of the Oscillator.

5.1.1 Tube Mounting. The tube is mounted in a hole in the center wall between the two cavities (Figure 13), by lightly clamping its grid flange against this wall. The clamp is a circular washer slotted from the center to form flexible fingers. This washer is only 0.01-inch thick. It is reinforced by a heavier washer whose inner diameter clears the grid flange. Three screws secure the assembly to the center wall. The tube is properly positioned before these screws are tightened to avoid strain in the glass envelope. See Figure 15.

5.1.2 Feedback. Feedback is accomplished by one or more probes as needed. The probes are connected to the inner conductor of the cathode cavity and are a slide fit in teflon bushings which line holes in the center wall. The probes are cemented into polystyrene rods which extend through the end of the cathode cavity. By means of these rods, the penetration depth of the probes into the plate cavity may be adjusted to control the feedback.

5.1.3 DC Connections. Since the tuning capacitors are placed in series with both plate and cathode inner conductors, dc current for

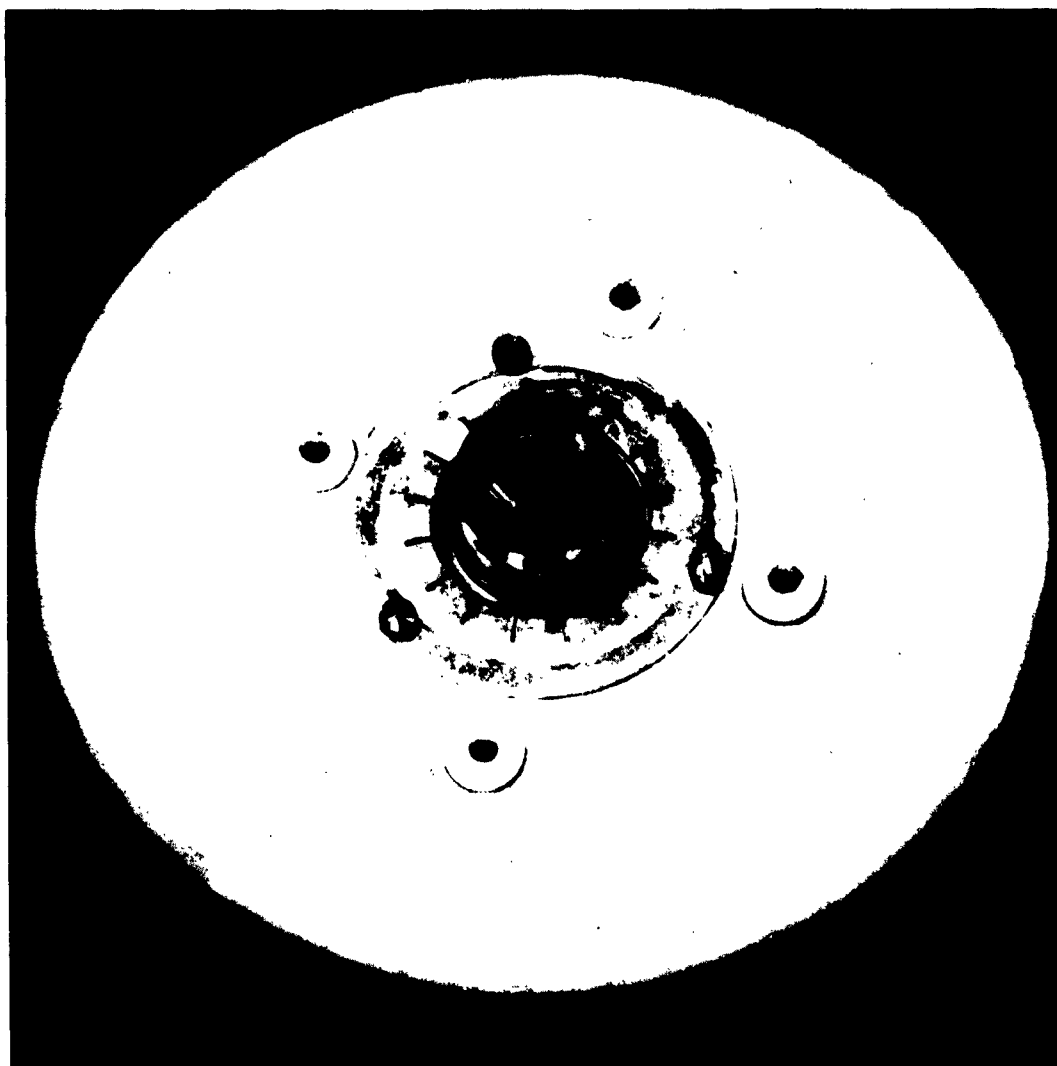


Figure 15

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Tube Mounting

5.1.3 - - Continued.

these electrodes must be carried by RF chokes. Three such chokes are required, one for the plate and two for the heater. The chokes are coaxial lines of high-characteristic impedance and designed to have a length equal to a quarter-wavelength at the low end of the band, since the best operation is expected in this region. Figure 16 shows a plan view of the chokes and inner conductor. The inner conductor of each choke is attached at one end to the appropriate tube electrode and bypassed to the outer conductor at the other end. The bypass capacitance is so large that it creates a near short circuit for RF. Since the short circuit is between $1/8$ and $3/8$ wavelength away from the cavity's center conductors at frequencies in the design band, they reflect a high impedance onto the center conductor over the entire band.

5.1.4 Ferroelectric Mounting. The ferroelectric tuning elements are mounted in cartridges with screw connections as shown in Figure 6. The screw connections mate with corresponding tapped holes in the center conductors as shown in Figure 17. The series inductance of the cartridge is about 6 to 10 m μ h.

5.1.5 Tuning Voltage Connections. Figure 17 shows a cross-section of the dc connection for applying tuning potentials to a ferroelectric in the inner conductor of a coaxial cavity. An insulated wire brings the tuning voltage to the end cap connecting to the cartridge, which is insulated from the remainder of the inner conductor by a ceramic ring capacitor. The center wire passes through the center of the ring and is insulated from its outer conductor by a dielectric which fills the intervening space. The end cap and cavity center conductor are both soldered to silvered surfaces of the ring capacitor. High voltage breakdown in the air at the outer surface of the ring capacitor is prevented by coating the surface with an insulating plastic.

The dielectric constant of the ceramic used in the ring is so high that its capacitive reactance is negligible in the 500 to 1000 Mc range. Exactly similar dc feed-through connections are used to supply the tuning voltage to ferroelectrics in both plate and cathode cavities. The dc feed-through arrangement for the quarter-wavelength chokes is also very similar.

5.1.6 Bench Test Setup. The arrangement of Figure 18 was used in testing the oscillator. Note that the plate tuning voltage supply floats on the plate voltage supply and requires isolation. Power and frequency measurements were made alternately. They could have been made simultaneously if a directional coupler had been substituted for the switch in the power output line.

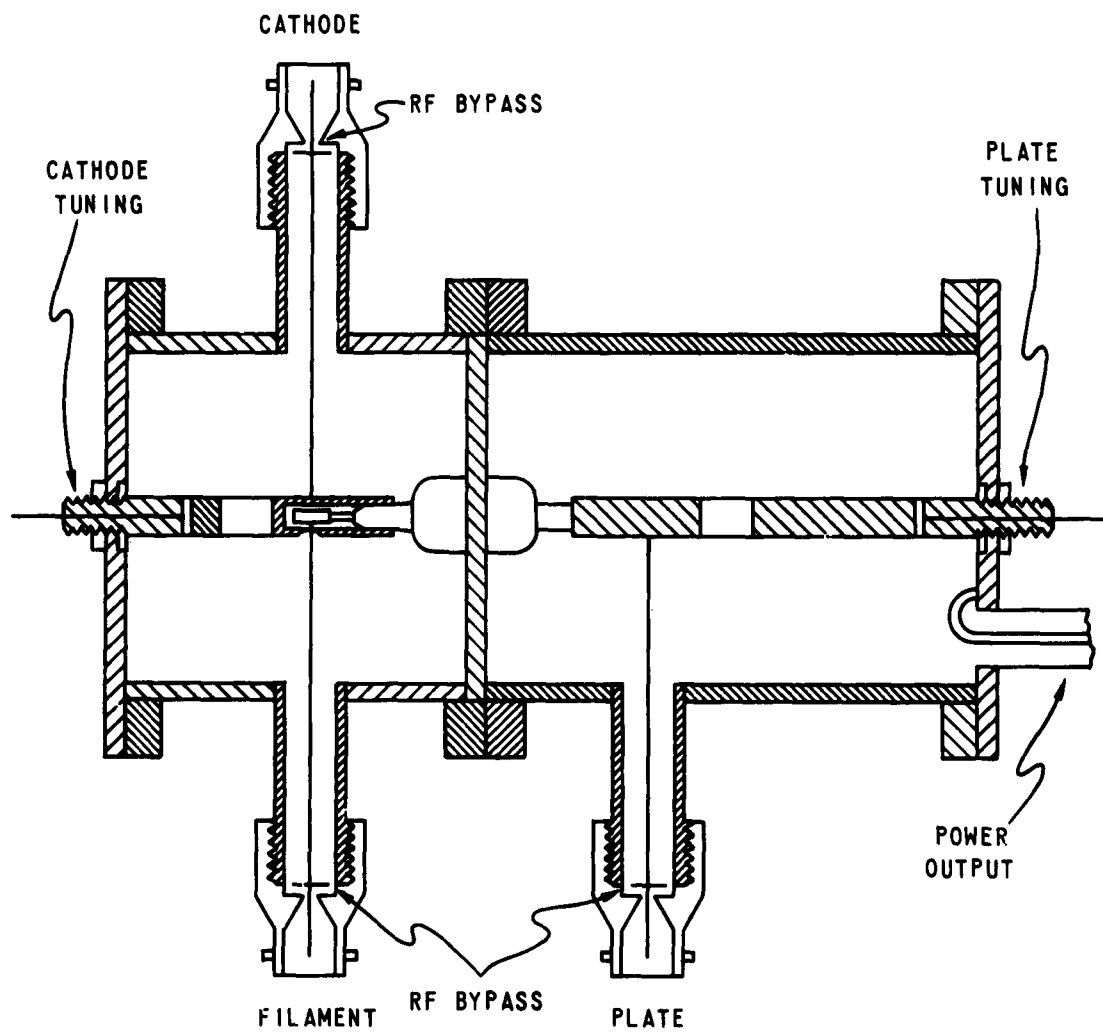


Figure 16
DC Connections

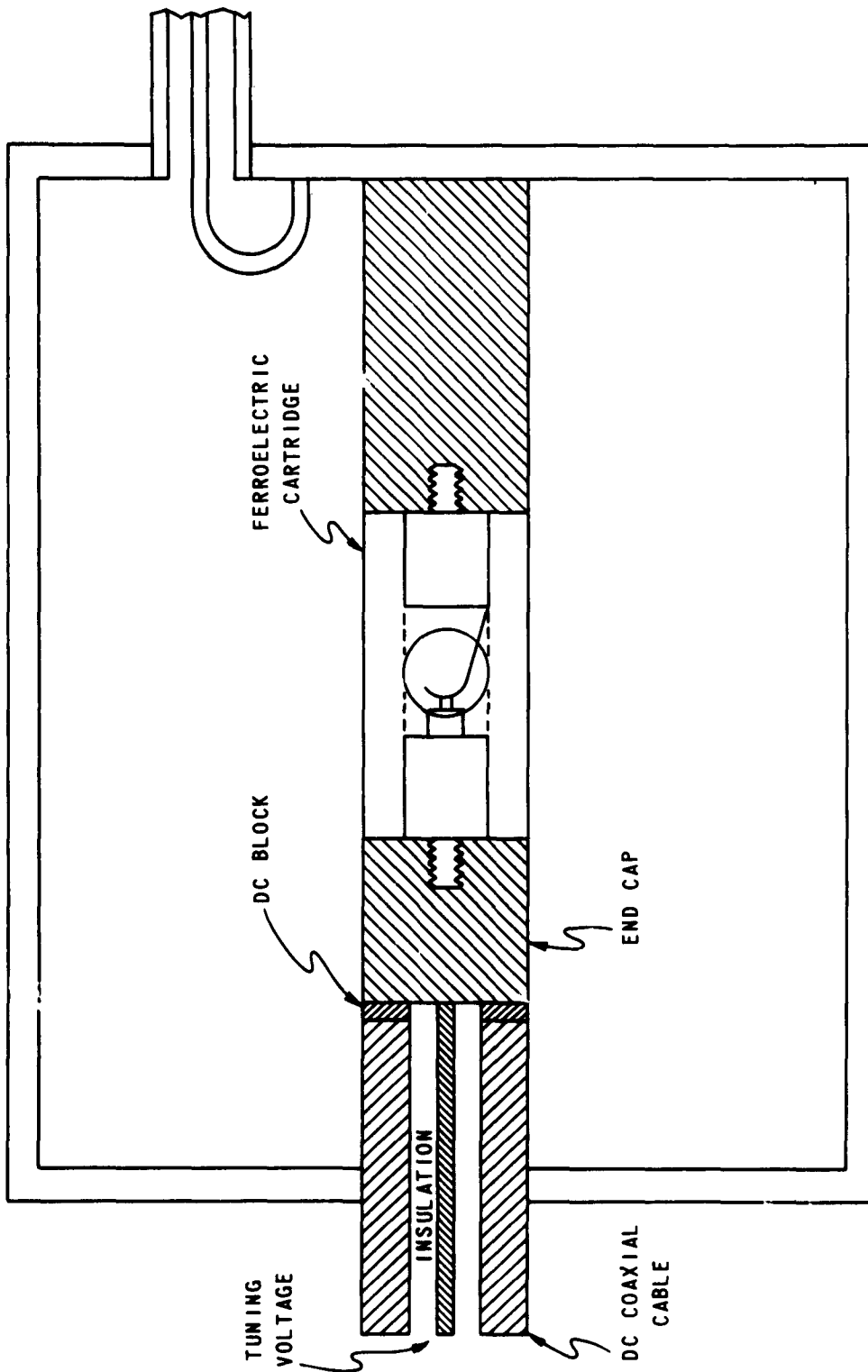


Figure 17
Method of Tuning a Microwave Cavity Electrically

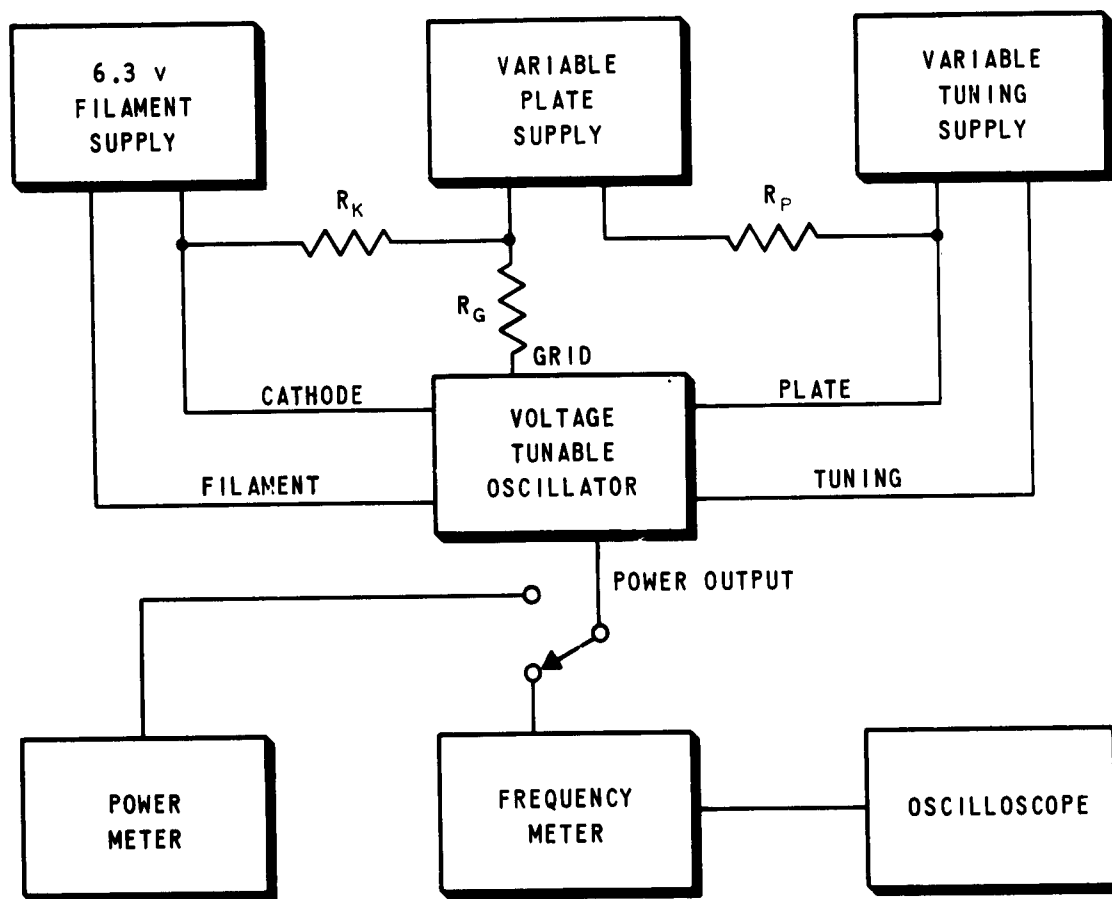


Figure 18
Bench Test Circuit

5.2 Expected Performance.

It was expected that the performance of the voltage tunable oscillator would depart from the ideal. Corrections to the ideal operation must be made to allow for:

1. the ferroelectric losses,
2. the temperature dependence of the dielectric constant of the ferroelectric tuning elements,
3. the series inductance of the tuning cartridge,
4. the reactances of the quarter-wavelength chokes, and
5. the variation in output impedance match over the band.

5.2.1 Ferroelectric Losses. In Section 3.3, the circuit Q of a series-tuned tank was calculated as a function of the Q of the tuning capacitor. The data presented in Section 3.1 shows that at room temperature, the average Q over a bias range of 0 to 40 kv/cm is about 12 for Glenco K9000. Neglecting ferroelectric heating for the moment, if Glenco K9000 is used to tune the oscillator and its average Q is used in the design calculations, the unloaded circuit Q , the power-matched voltage gain, and the power output into a matched load should be as shown in Figures 19 and 20 over the theoretical tuning range of this element.

5.2.2 Ferroelectric Heating. The RF power is lost as heat in the ferroelectric raises its temperature. The increase in temperature increases the Q somewhat but decreases the controllable range of the dielectric constant substantially. At temperatures above about 120° C, the dielectric constant decreases, thereby reducing the circuit Q , dissipating more heat in the ferroelectric, and heating the ferroelectric to higher temperatures. The power in the plate circuit should be kept low to avoid overheating the ferroelectric. This restriction limited maximum plate voltage to about 70 volts, and the oscillator output power to a few milliwatts.

5.2.3 Series Inductance of the Ferroelectric Cartridge. The cartridges in which the ferroelectric tuning elements are mounted have stray series inductance of between 6 and 10 mμh. The reactance which the series inductance introduces into the circuit must be overcome by reducing the size of the ferroelectric element. At 500 Mc, the ferroelectric must have about 20 to 30 ohms more reactance to compensate for the stray inductance. Since the circuit Q is proportional to the tuning capacitance, the stray inductance has the effect of reducing the circuit Q .

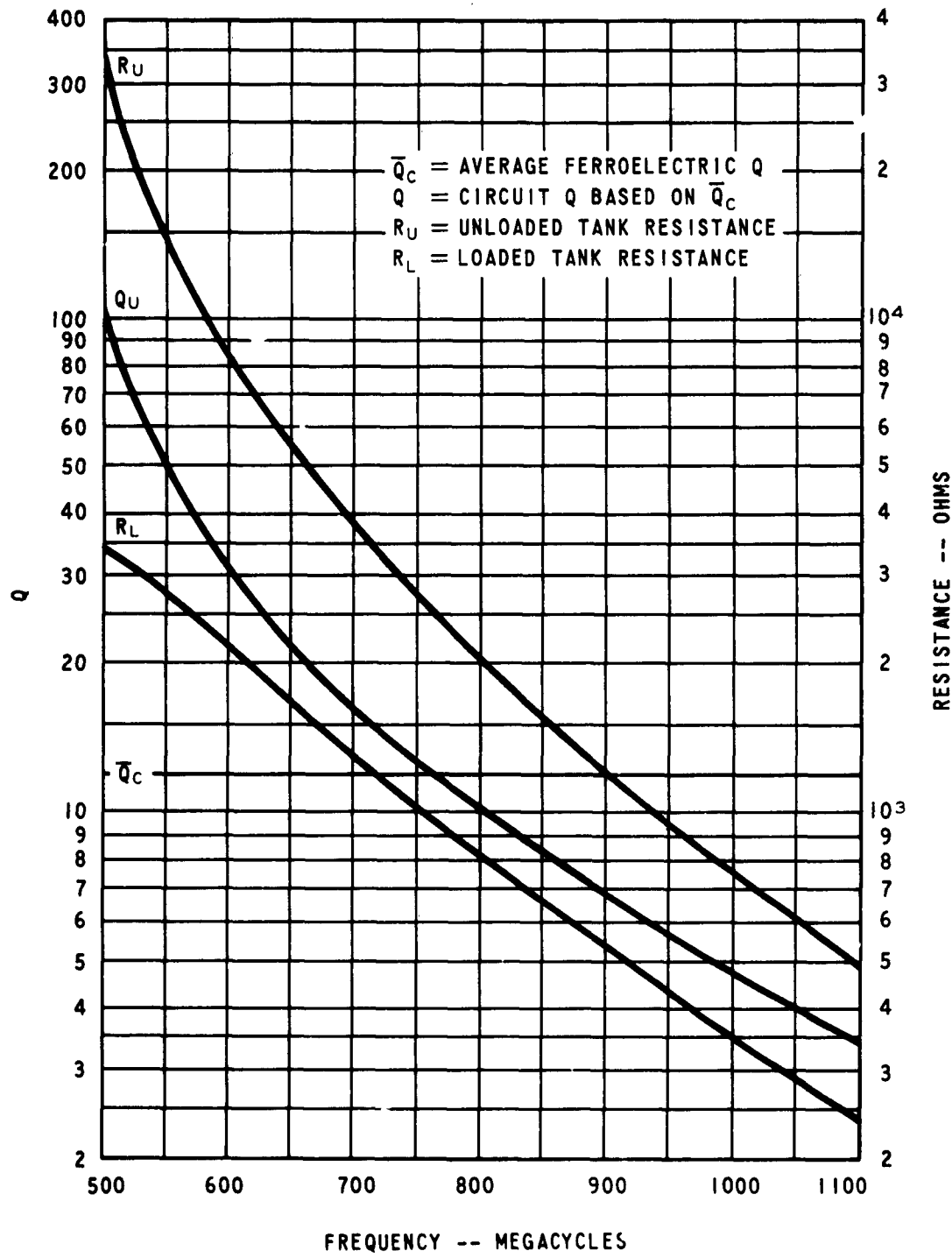


Figure 19

Average Plate Tank Parameters

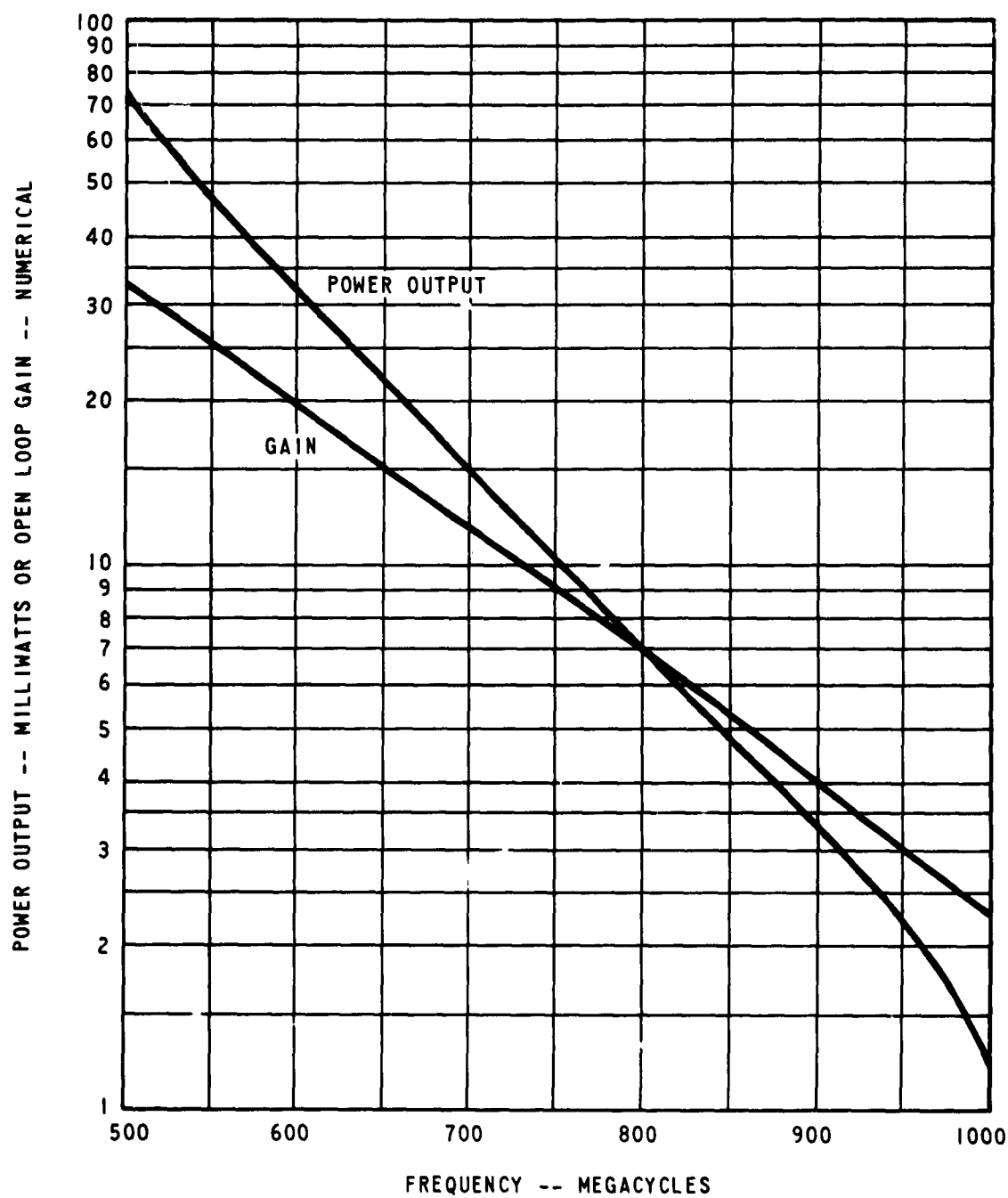


Figure 20

Performance of the Oscillator with Matched Load

5.2.4 Quarter-Wavelength Chokes. The tuning curve of Figure 14 was calculated without considering the effects of the quarter-wavelength chokes on the off-resonant frequency reactances of the tank circuits. Since the oscillator was expected to work best at low frequencies, the chokes were designed to be a quarter-wavelength at the low-frequency end of the band. At the high end, they are half-wavelength short circuits, and short out the tuning capacitor. We therefore expect the tuning curve to depart from the ideal above about 900 to 1000 Mc.

5.2.5 Variation of Output Impedance Match. The CW power output is limited to a few milliwatts by the considerations of Section 5.2.2. The fraction of this power which arrives at the load decreases with increasing frequency because the Q of the plate tank circuit decreases, more power is lost in the ferroelectric, and the equivalent generator impedance of the plate tank decreases thereby causing the load to be a mismatch.

5.3 Experimental Results.

5.3.1 Fixed Capacitor Tuning.

5.3.1.1 Frequency Range. The theoretical tuning curve was checked by substituting several sizes of teflon dielectric fixed capacitors for the plate tuning element. The resonant frequency was measured in each case. After a correction for the series inductance of the teflon capacitor leads, the frequency and effective tuning capacitance were plotted with the theoretical curve in Figure 14. The correspondence between the theoretical curve and the experimental points is seen to be quite close.

5.3.1.2 Power Output. At each frequency tuned by the fixed teflon capacitors, the output power was measured at four values of plate voltage. The curves of Figure 21 show that the maximum power output is approximately 100 mw across the band, as expected. The output power is reduced at the lower plate voltages approximately in proportion to the square of the plate voltage.

5.3.1.3 Heating. The heating effects of the tube were measured by mounting dummy cartridges in the plate and cathode cavities. The experimental arrangement was identical to the normal operation of the oscillator in all respects save two; there were no ferroelectric tuning elements in the cartridges and the thermocouples were attached to the cartridges to measure their temperature. Because the thermocouple

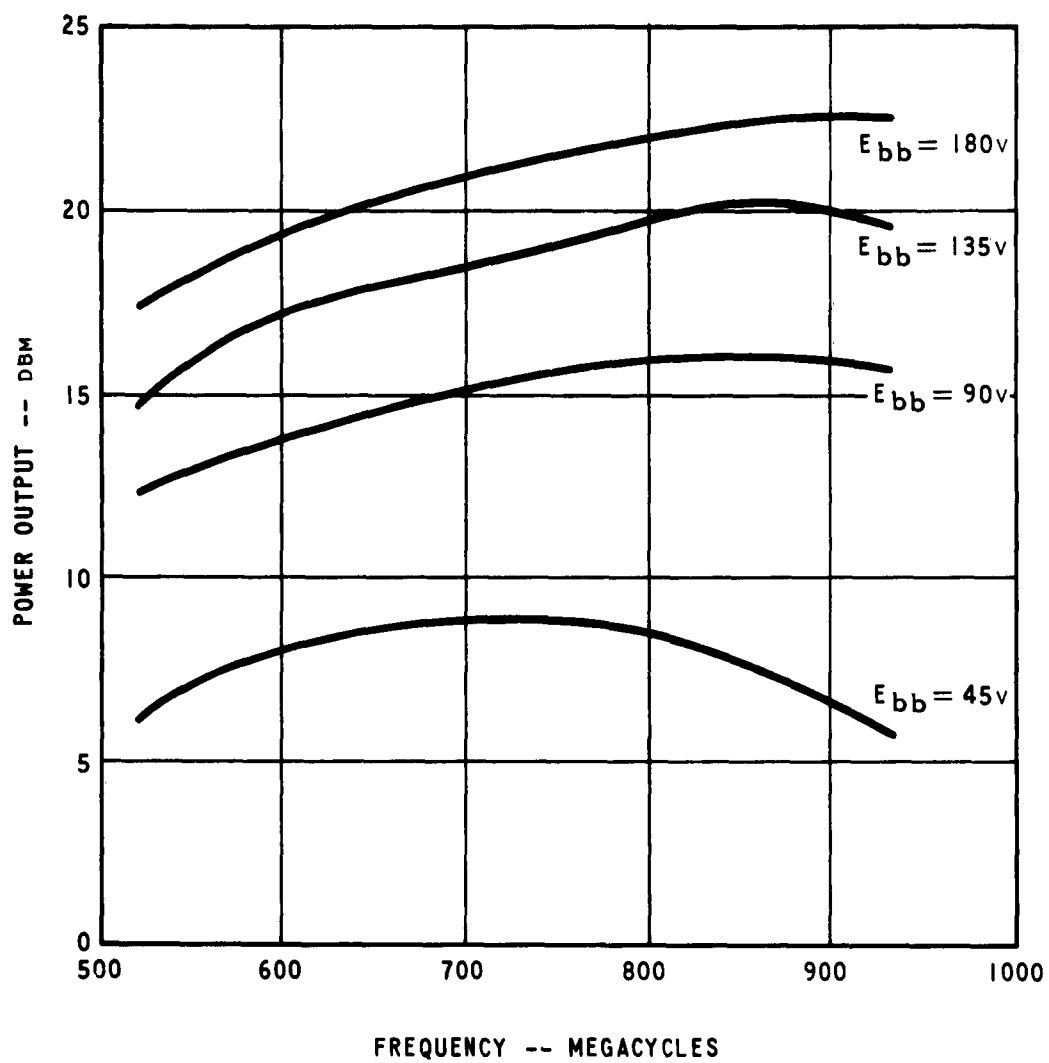


Figure 21
Oscillator Power Output

5.3.1.3 - - Continued.

leads were placed very close to the metal cavity walls, they had no measurable effect upon the oscillator's characteristics. The temperature rose to an equilibrium value of 34°C above room temperature in the plate cavity and 33.8°C above room temperature in the cathode cavity.

5.3.2 Ferroelectric Tuning. Figure 22 shows the tuning voltage and frequency curve for a Glenco K9000 capacitor with a zero voltage capacitance of about 11 pf. A short circuit was used in the cathode cavity so that the frequency is controlled by only one voltage. The capacitance range corresponding to this frequency range is also shown. From the tuning data on Glenco K9000 at 500 Mc, the approximate temperature for each value of tuning voltage may be estimated. The temperature curve is given in Figure 23.

The Q of the ferroelectric may be estimated from the curves of Section 3.1. Figure 22 shows the estimated ferroelectric Q from which the circuit Q is calculated according to the formulas of Section 3.3.3. The equivalent resistance across the tank corresponding to the ferroelectric loss is just Q times the grid-plate reactance. This resistance is shown in Figure 24. If the parallel equivalent resistance of the load and feedback circuits is about 30,000 ohms, then the expected power output curve would be as shown in Figure 24. The experimental points show that the circuit operates about as expected.

6. CONCLUSIONS.6.1 Summary of Oscillator Performance.

<u>Tuning Potential</u>	<u>Frequency</u>	<u>Power Output</u>
0 volts	491 Mc	3.5 mw
1000 volts	573 Mc	1.0 mw

6.2 Specifications.

On the basis of the work reported, the following specifications could be met:

1. An oscillator of this type could be built at any frequency up to 3 kMc.
2. The oscillator could be mechanically tuned over an octave band.

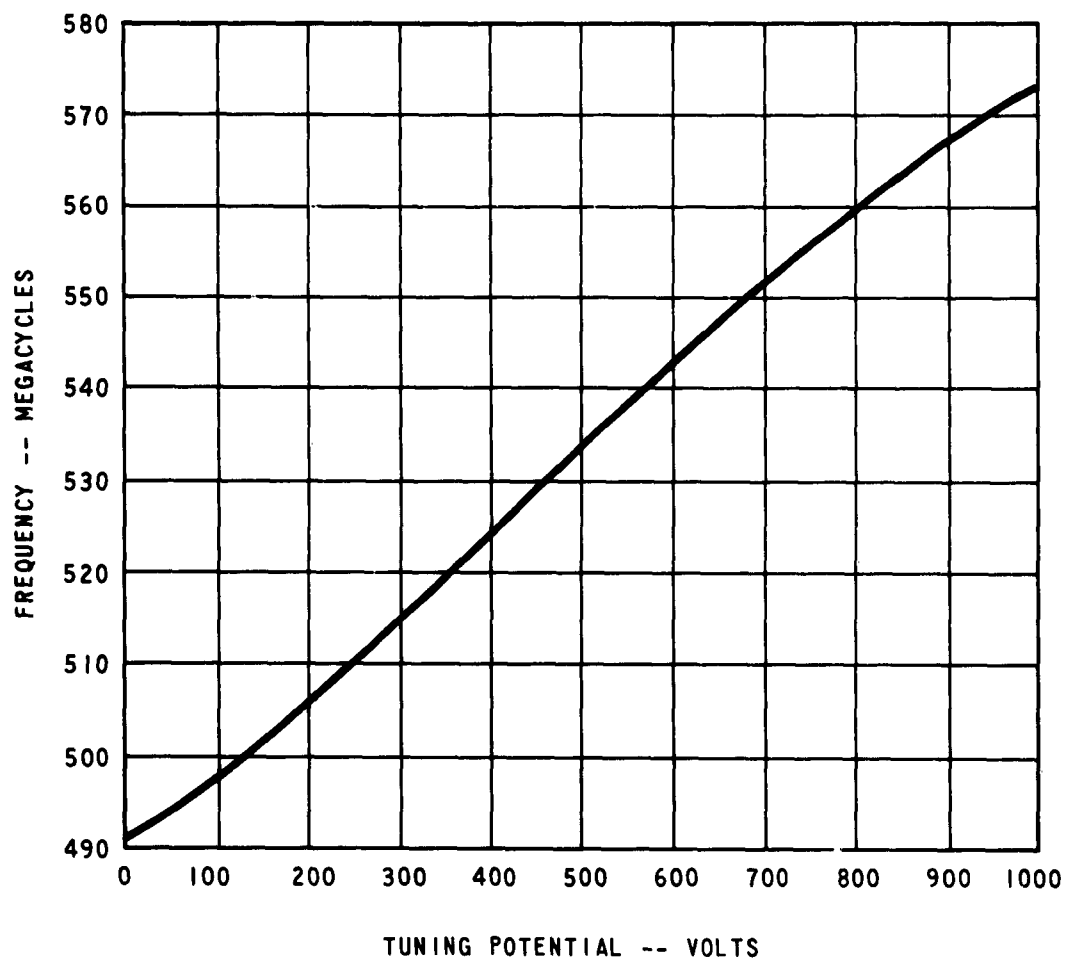


Figure 22

Ferroelectric Tuning

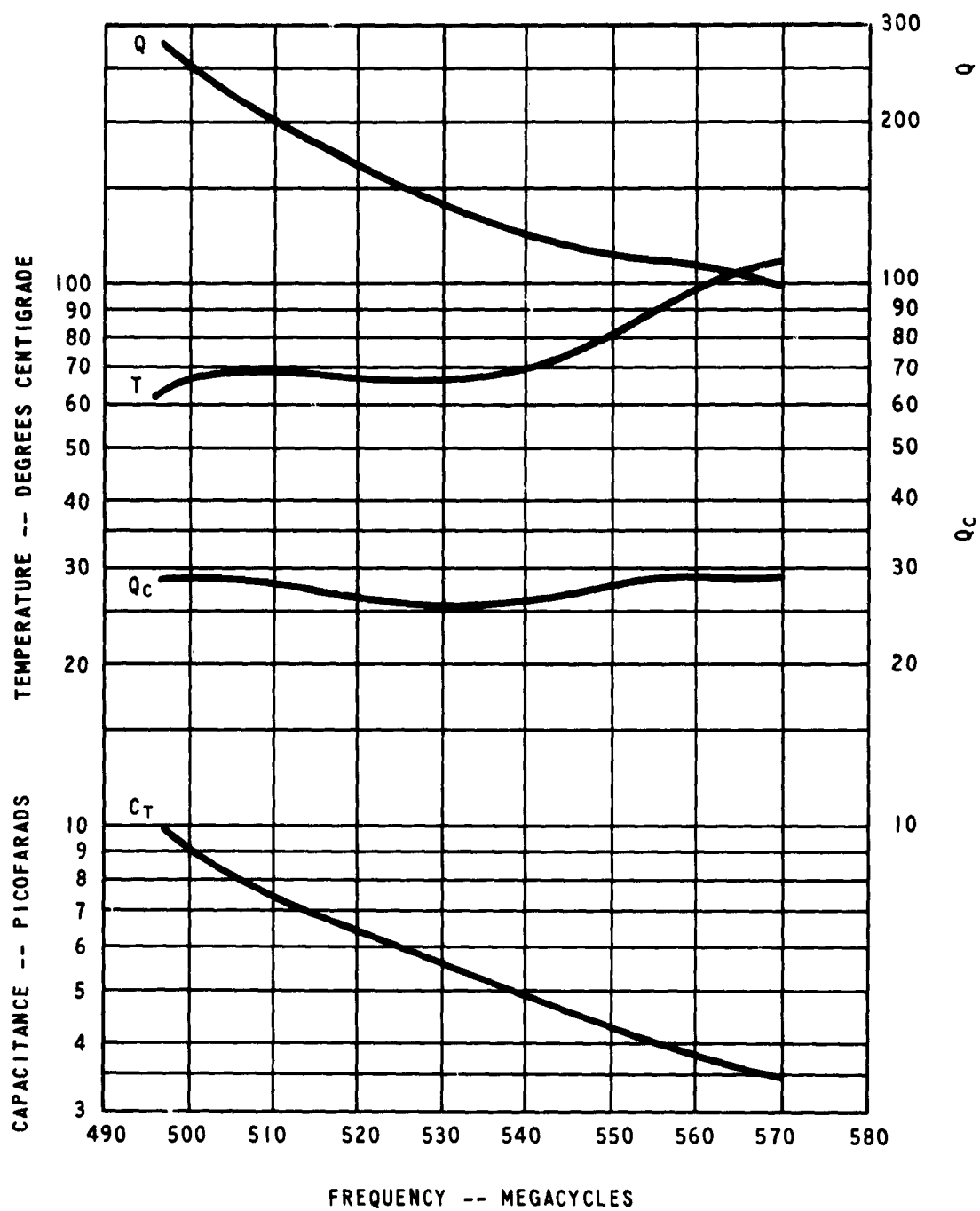


Figure 23

Ferroelectric Tuning Parameters

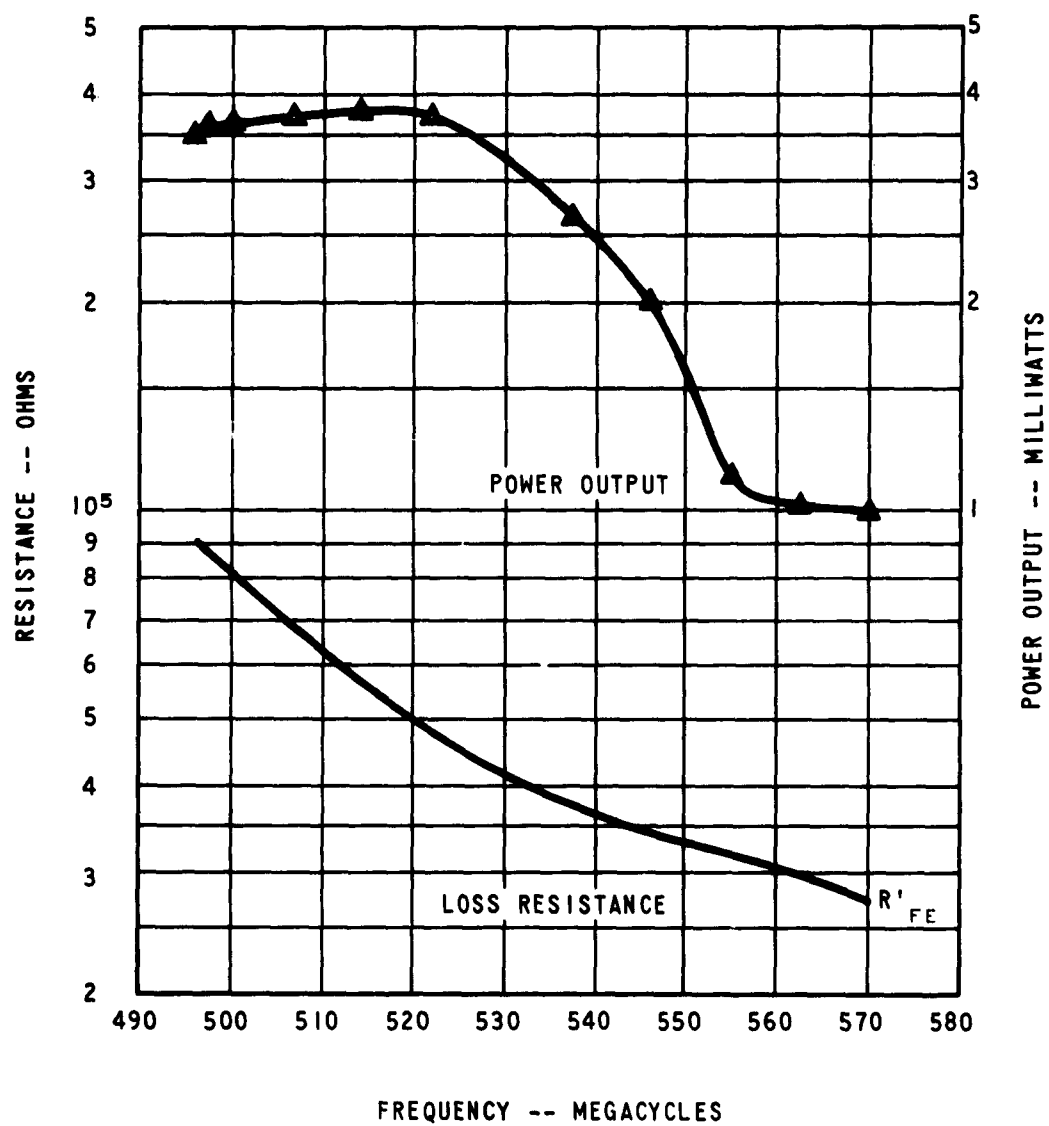


Figure 24

Oscillator Power Output and
Equivalent Ferroelectric Loss Resistance

6.2 - - Continued.

3. The oscillator could be electrically tuned over any 15 per cent portion of the mechanically-tuned octave.
4. The oscillator power output would be between 1 and 10 mw at all frequencies in the band.

6.3 Uses.

The ferroelectrically-tuned oscillator does not have the wide range and high output of mechanically-tuned oscillators. However, a 15 per cent tuning range is adequate for many applications. The frequency can be varied over this range band at megacycle rates. Some possible applications are:

1. laboratory FM signal source,
2. FM oscillator for communications or data transmission,
3. automatic frequency control systems,
4. electrical bandspread of wide-range mechanically-tuned oscillators,
5. receiver local oscillator for certain applications.

6.4 Improvements.

Many improvements could be made in the performance of this oscillator, by using a different tube and/or a different ferroelectric.

6.4.1 Improved Tubes. One of the difficulties with the present oscillator is that the tube envelope is large compared to the resonant lengths of line required for L-band operation. Small tubes of metal and ceramic construction have recently become available which will permit the ferroelectric to be mounted very close to, and in parallel with, the grid-plate gap.

A circuit of this type would have higher Q than the series-tuned circuit used in the work reported here. As a result, a wider tuning range and higher power output would be obtained.

6.4.2 Improved Ferroelectrics. The power output and tuning range of the oscillator have been shown to depend critically on the Q of the ferroelectric tuning element. A strong research effort, comparable to

6.4.2 - - Continued.

ferrite or semiconductor research, is needed to find or create materials with high Q or to discover any limitations which may exist. Work is also needed to understand the nature of ferroelectricity and to improve materials in other respects.

6.4.3 Possible Future Specifications. As an example of what could be done, let us consider the following specifications as a goal:

Tube Specification:

Design	Planar Triode
Plate Resistance	4000 ohms or less
Transconductance	10,000 micromhos
Grid-plate Capacitance	1 pf or less
Weight	4 oz. or less
Size	1 cu. in. or less.

Ferroelectric Specification:

Dielectric Constant (average)	10 min. , 10,000 max.
Dielectric Variation (ratio)	7 to 1 or greater
Loss Tangent	0.002 or less
Frequency	Specifications to apply to the frequency range of interest.

The specifications of an oscillator using such a tube and ferroelectric would be:

Oscillator Specification:

Tuning Range	:	Octave or greater
Power Output	:	Minimum 50 mw
Tuning	:	Electrical
Weight	:	1 lb. or less
Size	:	20 c. in. or less.

Gradual improvements in oscillator performance will come with improvements in tubes and ferroelectrics.

6.4.3 - - Continued.

The experiments with teflon capacitors show that if the losses in ferro-electrics can be reduced substantially, a wide range oscillator or amplifier can be built.

6.5 Acknowledgment.

The author gratefully acknowledges the assistance of Mr. R. Stewart who has done practically all of the experimental work including the construction of the tuning elements.

7. APPENDIX A. Ferroelectric Heating.

The heat equation for the case where heat is generated at the rate, $R \text{ cal/cm}^3/\text{sec}$, throughout the volume of the ferroelectric is

$$\frac{dT}{dt} = \frac{k}{c\delta} \frac{d^2T}{dX^2} + \frac{1}{c\delta} R \quad (A1)$$

where k = heat conductivity of the ferroelectric

c = heat capacity of the ferroelectric

δ = density of the ferroelectric

T = temperature rise above ambient in $^{\circ}\text{K}$

t = time

X = distance in ferroelectric from one end post.

At equilibrium, the temperature is no longer changing with time, i. e. ,

$$\frac{dT}{dt} = 0 = \frac{k}{c\delta} \frac{d^2T}{dX^2} + \frac{1}{c\delta} R. \quad (A2)$$

A common factor of $1/c$ may be dropped from the remaining terms. The resultant differential equation,

$$\frac{d^2T}{dX^2} = \frac{-R}{K} \quad (A3)$$

has a general solution

$$T = \frac{-R}{2K} X^2 + ax + b. \quad (A4)$$

Under the boundary conditions that $T = 0$ at the ends of the ferroelectric, values may be calculated for a and b

$$a = \frac{R}{2K} \quad b = 0 \quad (A5)$$

the solution is therefore

$$T = \frac{RX}{2K} (l - x), \quad \text{where } l \text{ is the length of the ferroelectric.} \quad (A6)$$

7. Appendix A - - Continued.

The temperature rise has a maximum value at the center of the ferro-electric where X has the value $l/2$. This maximum temperature rise, is

$$T_{\max} = \frac{R l^2}{8 k} . \quad (A7)$$

For barium titanate in which K has a value near $3 \cdot 10^{-3}$ (cal/sec) $(1/\text{cm}^2) (1/\frac{^\circ\text{C}}{\text{cm}})$, T_{\max} becomes

$$T_{\max} = \frac{R l^2}{24 \cdot 10^{-3}} = \frac{R l^2}{24} \cdot 10^3 \text{ } ^\circ\text{C}, \quad (A8)$$

APPENDIX B. The Q of a Series-Tuned Circuit.

In Figure 10, the impedance Z is

$$Z = j\omega L + \frac{R}{1 + j\omega R C}, \quad (B1)$$

which, after algebraic manipulation, may be written as

$$Z = \frac{R}{1 + Q_c^2} + j\omega L \left[1 - \frac{Q_c \cdot R}{(1 + Q_c^2) L} \right], \quad (B2)$$

$$\text{where } Q_c = \frac{B}{G} = R\omega C = \frac{K}{X_c}.$$

From the real and imaginary parts of this expression we define equivalent series circuit elements, R' and L' .

$$R' = \frac{R}{1 + Q_c^2} \quad L' = L \left[1 - \frac{Q_c R}{(1 + Q_c^2)\omega L} \right] \quad (B3)$$

The Q of the circuit may then be found as

$$Q = \frac{X_L}{R} = \frac{L'}{R'} = \frac{\omega L \left[1 - \frac{Q_c R}{(1 + Q_c^2)\omega L} \right]}{\frac{R}{1 + Q_c^2}} \quad (B4)$$

$$= \frac{\omega L}{R} (1 + Q_c^2 - Q_c \frac{R}{\omega L}) \quad (B5)$$

$$= (1 + Q_c^2) \frac{\omega L}{R} - Q_c \quad (B6)$$

For values of Q above about 4, 1 may be neglected compared to Q_c^2 . If this is done, the substitution $R/C = Q_c$ yields a simplification.

$$Q = R^2 \omega^2 C^2 \frac{\omega L}{R} - Q_c \quad (B7)$$

APPENDIX B, Cont'd.

By factoring out Q_c , we see that

$$Q = Q_c (\omega^2 LC - 1). \quad (B8)$$

The product LC may be replaced by $1/\omega_0^2$ defined by the equation

$$\omega_0^2 = \frac{1}{LC}. \quad (B9)$$

Then,

$$Q = Q_c \left(\frac{\omega^2}{\omega_0^2} - 1 \right). \quad (B10)$$

The frequencies, ω and ω_0 , may be related to the stray and tuning capacitances through the reactance formulae. At the frequency f ,

$$X_c = \frac{1}{\omega C} \quad X_L = \omega L \quad (B11)$$

and at the frequency f_0 ,

$$X_{c0} = \frac{1}{\omega_0 C} \quad X_{L0} = \omega_0 L. \quad (B12)$$

These equations may be solved for X_{c0} and X_{L0} which are then given by the formulas

$$X_{c0} = \frac{\omega}{\omega_0} X_c \quad X_{L0} = \frac{\omega_0}{\omega} X_L \quad (B13)$$

The frequency, f_0 , is defined as the series LC resonant frequency.

Therefore, at f

$$X_{c0} = X_{L0} \quad (B14)$$

or,

$$\frac{\omega}{\omega_0} X_c = \frac{\omega_0}{\omega} X_L. \quad (B15)$$

The useful form of this equation is

$$\frac{\omega^2}{\omega_0^2} = \frac{X_L}{X_c}. \quad (B16)$$

APPENDIX B, Cont'd.

The X_C in this formula is the reactance of the tuning capacitor. At any resonant frequency of the circuit, X_L , is equal to the sum of the stray capacitive reactance, X_{c_s} , and the tuning capacitive reactance, X_{c_T} .

That is,

$$X_L = X_{c_s} + X_{c_T}$$

$$X_c = X_{c_T}$$

Therefore,

$$\frac{\omega^2}{\omega_0^2} = \frac{X_{c_s} + X_{c_T}}{X_{c_T}} = 1 + \frac{X_{c_s}}{X_{c_T}} = 1 + \frac{\frac{1}{\omega C_s}}{\frac{1}{\omega C_T}} \quad (B17)$$

$$= \frac{C_T}{C_s} + 1. \quad (B18)$$

When this expression is substituted into the formula for circuit Q, the expression is simplified to

$$Q = Q_{c_T} \frac{C_T}{C_s}. \quad (B19)$$

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